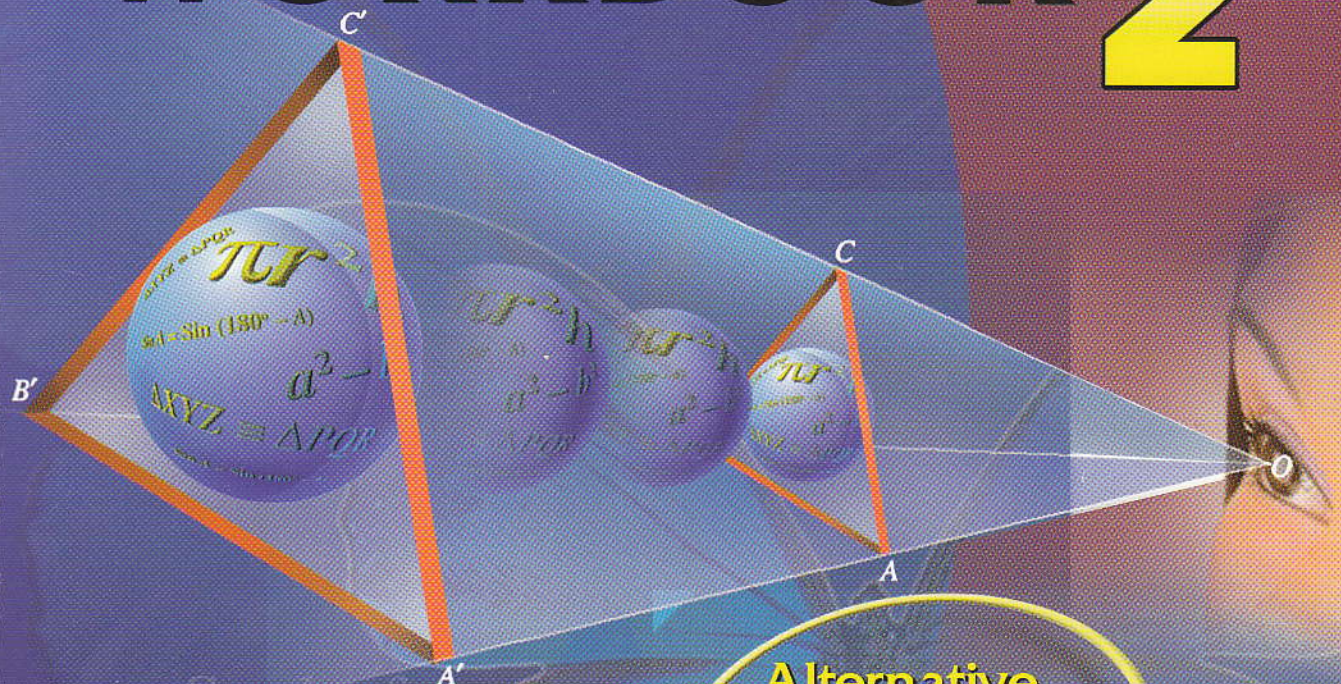
shinglee

New Syllabus

MATHEMATICS WORKBOOK 2



Alternative Assessment & CD Included

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CONTENTS

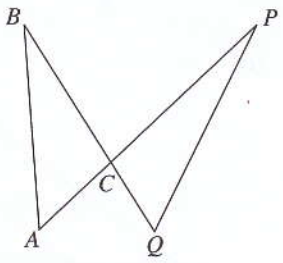
Chapter 1 Congruence and Similarity

Summary

1. Congruent figures or objects have exactly the same shape and the same size.
2. A figure and its image under translation, rotation or reflection are congruent.
3. Similar figures or objects have exactly the same shape but not necessarily the same size.
4. Two polygons are similar if
   (a) all the corresponding angles are equal, and
   (b) all the ratios of the corresponding sides are equal.
5. Congruence is a special case of similarity.
6. A figure and its image under an enlargement are similar.
7. An enlargement with a scale factor greater than 1 produces an enlarged image. An enlargement with a scale factor between 0 and 1 produces a diminished image. An enlargement with a scale factor of 1 produces a congruent image.
8. If the linear scale of a map is $1 : x$, it means that 1 cm on the map represents x cm on the actual piece of land.

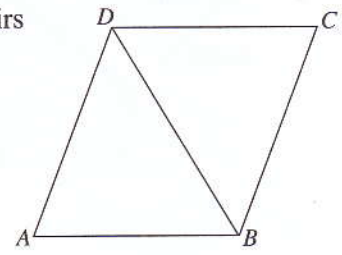
Practice Questions

1. Given that $\triangle ABC$ is congruent to $\triangle QPC$, copy and complete the following.

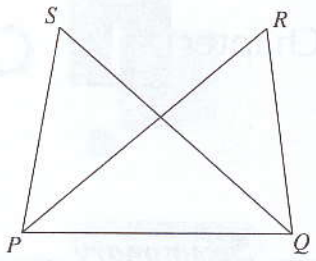


   (a) $AB =$ _____, **(b)** $BC =$ _____,
   (c) $CQ =$ _____, **(d)** $A\hat{B}C =$ _____,
   (e) $C\hat{Q}P =$ _____, **(f)** $P\hat{C}Q =$ _____.

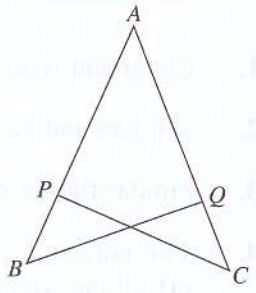
2. Given that $\triangle ABD$ is congruent to $\triangle CDB$, state six pairs of corresponding equal parts.



3. Given that $\triangle PQS$ is congruent to $\triangle QPR$, state six pairs of corresponding equal parts.

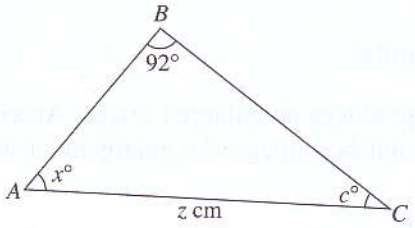


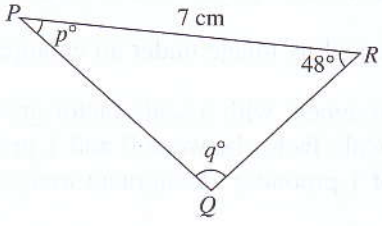
4. Given that $\triangle ABQ$ is congruent to $\triangle ACP$, write down six pairs of corresponding equal parts.



5. In each of the following figures, $\triangle ABC$ is congruent to $\triangle PQR$ (not drawn to scale). Write down the value of each unknown.

(a)





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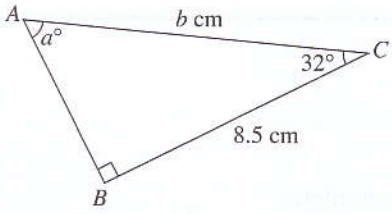

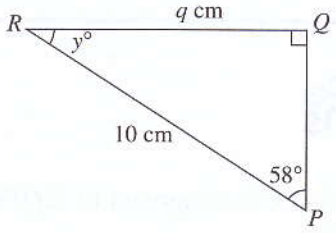

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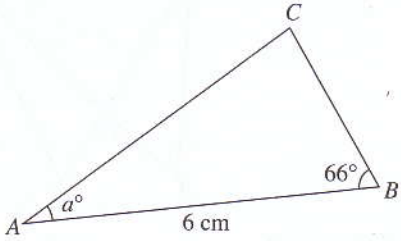

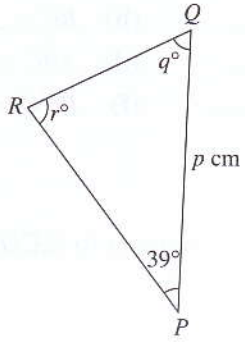

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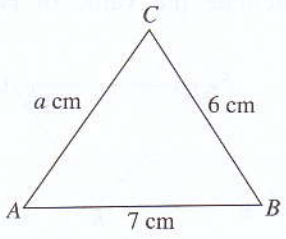
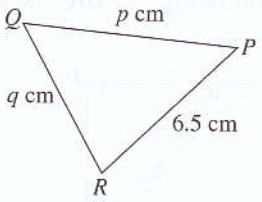
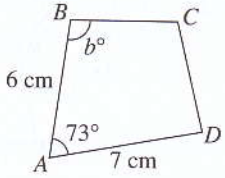
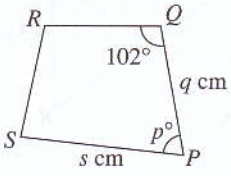




6. Each of the following pairs of figures are congruent (not drawn to scale). Find the value of each unknown.

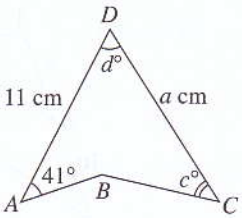
(a)

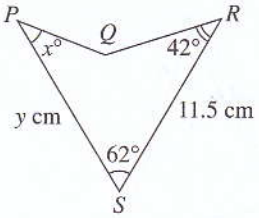




$ABCD \equiv PQRS$

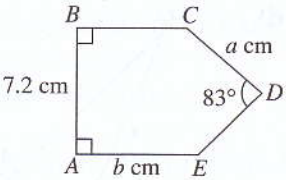
(b)

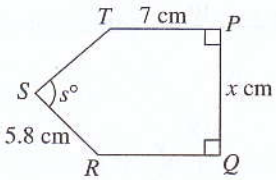




$ABCD \equiv PQRS$

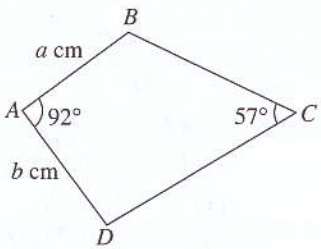
(c)

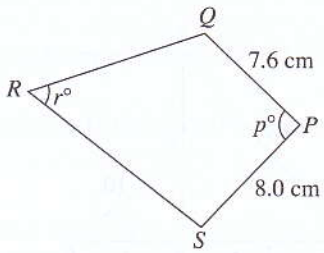




$ABCDE \equiv PQRST$

(d)

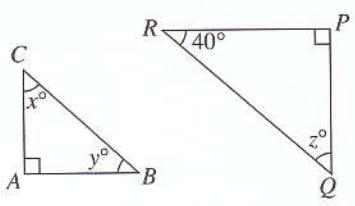




$ABCD \equiv PQRS$

7. In each of the following, $\triangle ABC$ is similar to $\triangle PQR$. Calculate the value of each of the unknowns.

(a)



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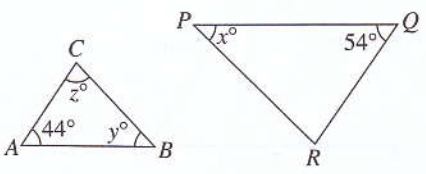

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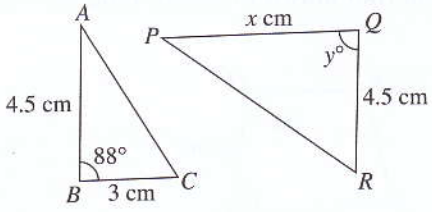

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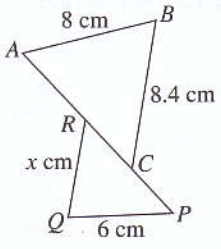
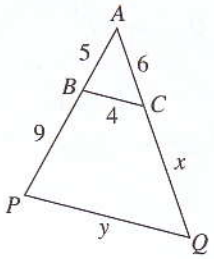


8. Given that $\triangle ABC$ is similar to $\triangle APQ$, calculate the value of each of the unknowns. The measurements in each figure are in cm.

(a)



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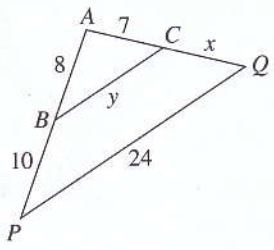

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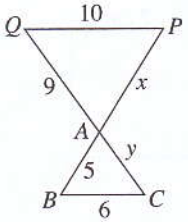

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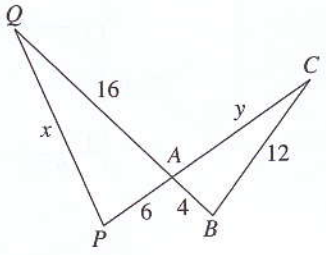

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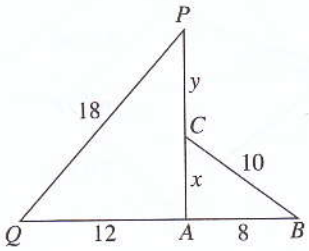

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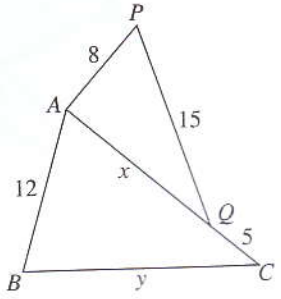

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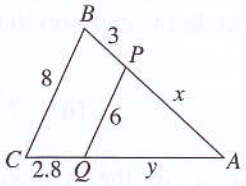
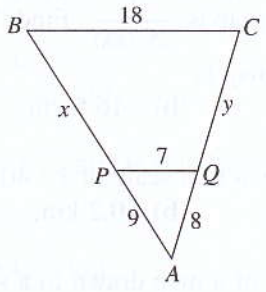
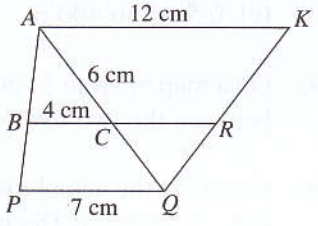


(h)



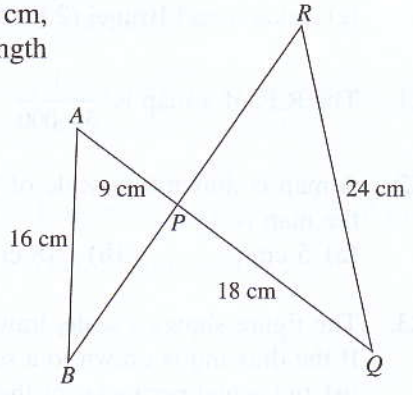
9. In the figure, $\triangle ABC$ is similar to $\triangle APQ$ and $\triangle QCR$ is similar to $\triangle QAK$. Given that $BC = 4$ cm, $PQ = 7$ cm, $AC = 6$ cm and $AK = 12$ cm, calculate the length of

(a) CQ, **(b)** CR.



10. In the figure, $\triangle PQR$ is similar to $\triangle PBA$. Given that $PQ = 18$ cm, $QR = 24$ cm, $AP = 9$ cm and $AB = 16$ cm, calculate the length of

(a) PB, **(b)** PR.



11. A map is drawn to a scale of 2 cm to 3 km. Find the actual distance, in km, between two towns if the distance apart on the map is

(a) 24 cm, **(b)** 10.5 cm, **(c)** 14.2 cm, **(d)** 2.6 cm.

12. Two cities are 480 km apart. What is their distance apart on the map drawn to a scale of

(a) 2 cm to 25 km, **(b)** 5 cm to 75 km, **(c)** 9 cm to 25 km, **(d)** 0.5 cm to 120 km?

13. The distance between two towns P and Q on a map is 16 cm. Find the actual distance, in km, if the scale of the map is

(a) 2 cm to 3 km, **(b)** 1.2 cm to 3 km, **(c)** 2.4 cm to 9 km, **(d)** 0.5 cm to 0.25 km.

14. On a scale drawing, the height of a school block is 4 cm. If the actual height of the block is 30 m, what is the scale of the drawing?

15. The distance between two car parks on a map with a scale of 5 cm to 2 km is 7 cm. What is the actual distance, in km, between these two car parks? What would be the distance, in cm, between these two car parks on another map drawn to a scale of 6 cm to 4 km?

16. The R.F. of a map is $\frac{1}{25\ 000}$. Find the actual distance, in km, between two villages if the distance apart on the map is

(a) 18 cm, **(b)** 16.5 cm, **(c)** 65 cm, **(d)** 7.4 cm.

17. A map is drawn to a scale of 1 : 40 000. What distance, in cm, on the map will represent

(a) 800 m, **(b)** 0.2 km, **(c)** 3.6 km, **(d)** 2 km 400 m?

18. Find the R.F. of a map drawn to a scale of

(a) 2 cm to 9 km, **(b)** 3 cm to 4.5 km,

(c) 0.5 cm to 400 m, **(d)** 7.5 cm to 105 km.

19. On a map of scale 1 : 500 000, the distance between two towns is 17.6 cm. Find the actual distance between the two towns in km.

20. Calculate the actual distance between the following places in km given that the scale of the map is 1 : 7 500 000. The map distances are given in brackets.

(a) Singapore and Jakarta (12 cm) **(b)** Singapore and Medan (8 cm)

(c) Jakarta and Brunei (20.5 cm) **(d)** Kuala Lumpur and Surabaya (22 cm)

21. The R.F. of a map is $\frac{1}{50\ 000}$. Find the area on the map which represents an area of 20 km^2.

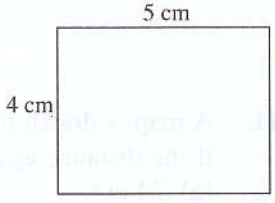
22. A map is drawn to a scale of 1 : 20 000. Find the actual area, in km^2, of a field whose area on the map is

(a) 5 cm^2, **(b)** 18 cm^2, **(c)** 75 cm^2, **(d)** 124 cm^2.

23. The figure shows a scale drawing of a rectangular piece of land. If the drawing is drawn to a scale of 1 cm to 15 m, find

(a) the actual perimeter of the land in m,

(b) the actual area of the land in m^2.



24. A plan of a shopping complex is drawn to a scale of 1 cm to 5 m. Express the scale of the plan in the form 1 : n. Find the perimeter and area of a rectangular shop space which measures 2.4 cm by 4 cm on the plan.

25. A map is drawn to a scale of 1 cm to 4 km. A forest has an area of 64 km^2. Find, in cm^2, its area on the map. What will be the area of the forest drawn on another map with a scale of 1 cm to 2 km?

26. Given that 4 cm on a map represents 3 km on the ground,

(a) calculate the actual distance, in km, between two towns which are 10.5 cm apart on the map,

(b) find the R.F. of the map,

(c) calculate, in cm^2, the area of a town council on a map given that its actual area is 32.4 km^2.

27. A park occupies an area of 24 cm^2 on a map. Find its actual area, in m^2, if the scale of the map is

(a) 2 cm to 15 m, **(b)** 4 cm to 25 m,
(c) 4 cm to 0.6 km, **(d)** 1.5 cm to 120 m.

28. The distance between two cities on a map with a scale of 1 : 1 500 000 is 14 cm. Find the distance between these two cities on a map with a scale of

(a) 3 cm to 70 km, **(b)** 4 cm to 35 km,
(c) 5 cm to 10.5 km, **(d)** 7 cm to 6 km.

29. The area of a field drawn on a map with a scale of 1 cm to $\frac{1}{2}$ km is 36 cm^2. What will be the area, in cm^2, of the field drawn on a map with a R.F. of

(a) 1 : 25 000, **(b)** 1 : 12 500,
(c) 1 : 75 000, **(d)** 1 : 200 000.

30. A map is drawn to a scale of 1 : 120 000.

(a) Calculate the actual distance, in km, represented by 5.4 cm on the map.
(b) Two towns are 10 km and 80 m apart. Calculate, in cm, their distance apart on the map.
(c) On the map, a lake has an area of 3.6 cm^2. Calculate, in km^2, the actual area of the lake.

31. A nature reserve of area 225 km^2 is represented on a map by an area of 36 cm^2. Find the R.F. of the map. What will the area of the nature reserve be on a map with a scale of 1 cm to 5 km?

32. A scale model of a building is made. Given that the area of a hall on the model is $\frac{1}{100}$ of the actual area, calculate the length of the hall on the model if its actual length is 40 m.

33. State whether each of the following statements is true or false. Give your reason(s) or use an example to explain your answers.

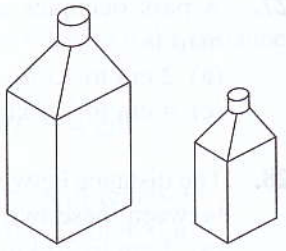
(a) All equilateral triangles are similar.
(b) All squares are congruent to one another.
(c) All circles are similar.
(d) Any two semicircles selected will be congruent to each other.
(e) The faces of a cube are congruent to one another.
(f) The face of a cube must be congruent to the face of another cube.
(g) The face of a rectangle is always similar to the face of another rectangle.
(h) All rhombuses are similar.
(i) All squares are similar.
(j) The diagonals of a parallelogram will bisect it into two congruent triangles.
(k) The diagonal of a kite will divide it into two congruent triangles.
(l) The angle bisector of a rectangle will divide it into two congruent triangles.

34. A water tank in a photograph is 8 cm long and 4 cm high. If the actual height of the tank is 3.2 m, find the actual length of the tank.

35. Two similar water containers are shown on the right. The radius of the smaller bottle is 1.2 cm and that of the larger bottle is 2 cm.

(a) If the height of the larger bottle is 8.4 cm, calculate the height of the smaller bottle.

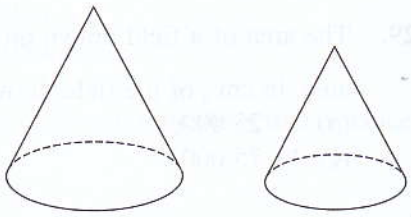
(b) If the length of the smaller bottle is 7.6 cm, calculate the length of the larger bottle.



36. Two similar cones are shown on the right. Given that the height of the larger cone is 24 cm and that of the smaller cone is 10 cm, calculate

(a) the radius of the larger cone if the radius of the smaller cone is 5.5 cm,

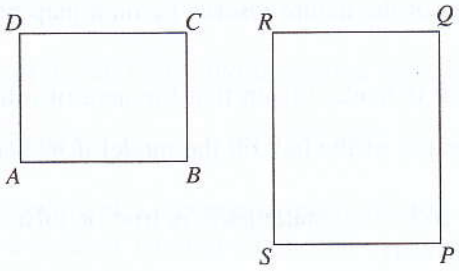
(b) the circumference of the smaller cone if that of the larger cone is 84 cm.



37. The length and width of a rectangle *ABCD* are 24 cm and 18 cm respectively. Given that rectangle *ABCD* is similar to rectangle *PQRS*,

(a) find the width of the rectangle, *PQRS*, if its length is 36 cm,

(b) find the length of the rectangle, *PQRS*, if its width is 36 cm.



Alternative ASSESSMENT

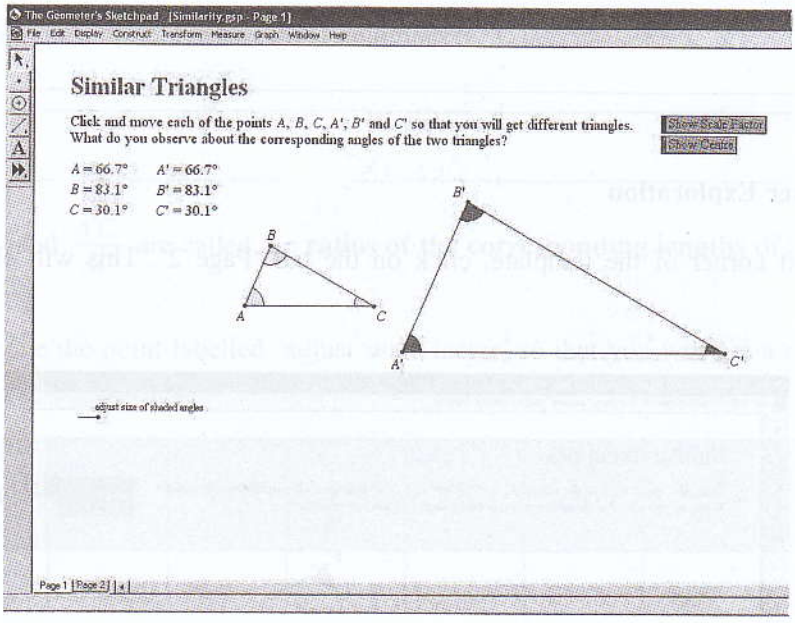
IT Worksheet: Properties of Similar Triangles

You need the Geometer's Sketchpad (GSP), a dynamic geometry software, to view and interact with the GSP template for this worksheet. If your school does not have a licensed copy of version 4, you may download the free evaluation version from www.keypress.com for trial first.

The purpose of this worksheet is to explore the properties of similar triangles.

Section A: Exploration

Open the appropriate template from the Workbook CD.



1. The template shows two triangles. What do you notice about the shapes of $\triangle ABC$ and $\triangle A'B'C'$? **[1]**

2. What do you notice about the angles of $\triangle ABC$ and the corresponding angles of $\triangle A'B'C'$? **[1]**
Note: Corresponding angles refer to $\angle A$ and $\angle A'$, $\angle B$ and $\angle B'$, and $\angle C$ and $\angle C'$.

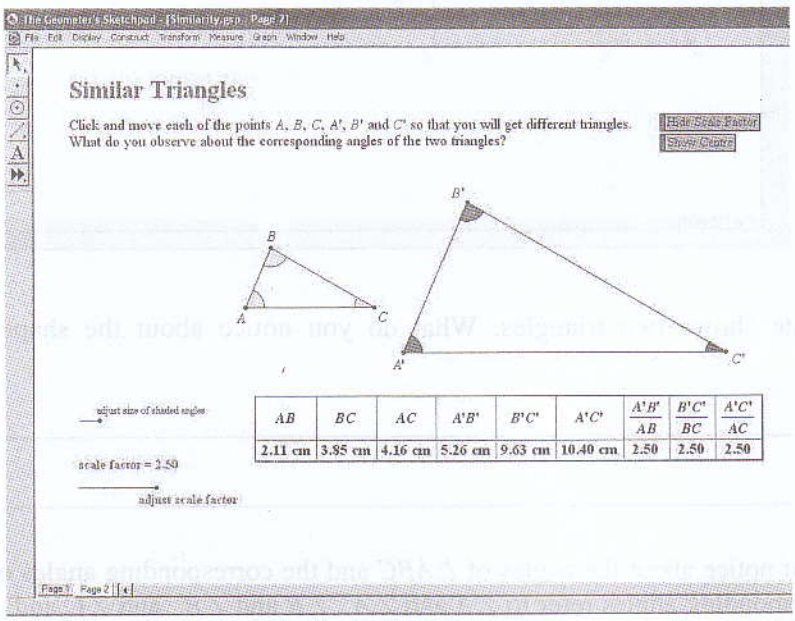
3. Click and move each of the points A, B, C, A', B' and C' so that you will get different pairs of triangles. What do you notice about the shapes of $\triangle ABC$ and $\triangle A'B'C'$? **[1]**

4. Click and move each of the points A, B, C, A', B' and C'. What do you notice about the angles of $\triangle ABC$ and the corresponding angles of $\triangle A'B'C'$? **[1]**

5. $\triangle ABC$ and $\triangle A'B'C'$ are called **similar** triangles. State two properties of similar triangles based on your exploration in this section. **[1]**

Section B: Further Exploration

At the bottom left corner of the template, click on the tab 'Page 2'. This will show the template below.



6. Click and move each of the points A, B, C, A', B' and C' so that you will get four different pairs of similar triangles. Complete the table below. [3]

No.	AB	BC	AC	$A'B'$	$B'C'$	$A'C'$	$\frac{A'B'}{AB}$	$\frac{B'C'}{BC}$	$\frac{A'C'}{AC}$
1									
2									
3									
4									

7. What do you notice about the last three columns? [1]

Note: $\frac{A'B'}{AB}$, $\frac{B'C'}{BC}$ and $\frac{A'C'}{AC}$ are called the **ratios of the corresponding lengths** of the two triangles.

8. Click and move the point labelled 'adjust scale factor' so that you will get a different value for the scale factor k. Repeat Q6 above and complete the table below. [3]

No.	AB	BC	AC	$A'B'$	$B'C'$	$A'C'$	$\frac{A'B'}{AB}$	$\frac{B'C'}{BC}$	$\frac{A'C'}{AC}$
1									
2									
3									
4									

9. What do you notice about the last three columns? [1]

10. Based on your exploration in this section, state one property of similar triangles. **[1]**

Section C: Animation

- Right-click on the table in the template and select: ***Add Table Data...***
- Select the second option: ***Add 10 Entries As Values Change, Adding 1 Entry Every 1.0 Second(s).*** Click ***OK***.
- Select the vertex *A* of the triangle and the point '***adjust scale factor***'.
- Choose from the Toolbar: ***Display*** ▸ ***Animate Point*** or ***Animate Objects***.

Section D: Conclusion

11. Write down one main lesson that you have learnt from this worksheet. **[1]**

Final Score: ☐ **/ 15**

Final Score	12–15	10–11	8–9	6–7	0–5
Grade	A	B	C	D	F

Teacher's Comments (if any):

Journal Writing: Congruence and Similarity

One of your classmates is confused over when two figures are considered to be congruent and when they are considered to be similar to each other. By providing some examples, explain to your classmate, clearly defining the differences between congruence and similarity.

Scoring Rubric:

Competency Level	Mathematical Concept	Mathematical Communication	Effort
4	Showed complete understanding of congruence and similarity	Gave clear and complete explanations and used accurate mathematical terminology	Provided a lot of good examples
3	Showed nearly complete understanding of congruence and similarity	Gave nearly complete explanations and/or made some minor errors in mathematical terminology used	Provided sufficient examples
2	Showed some understanding of congruence and similarity	Gave incomplete explanation and/or made some errors in mathematical terminology used	Provided limited examples
1	Showed limited understanding of congruence and similarity	Gave explanations which were difficult to understand and/or made major errors in mathematical terminology used	Provided unclear examples
0	Showed no understanding of congruence and similarity	Gave muddled explanations and did not use any accurate mathematical terminology	Provided no examples
Score			

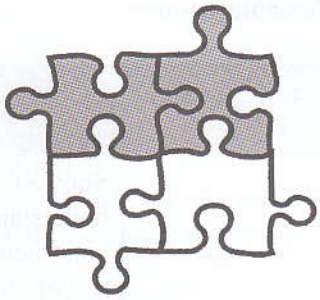
Final Score:

____ / 12

Final Score	10–12	8–9	6–7	4–5	0–3
Grade	A	B	C	D	F

Teacher's Comments (if any):

Performance Task: Tessellation



You are a budding artist aspiring to be as famous as M. C. Escher who is well known for his tessellation art pieces and illusion art. You should search the Internet (e.g. www.mcescher.com) to look at some of Escher's art pieces for some ideas of what a good and beautiful tessellation art piece is.

Tessellation is the art of placing all the congruent figures together so that there is no gap in between. Your task is to design your own master art piece using some congruent figures that you will craft. You can either use manual drawing or computer technology (you can search the Internet for some free easy-to-use tessellation software). Whatever tool you use, remember to colour your tesellation to make it more attractive.

Scoring Rubric:

Competency Level	Mathematical Concept	Creativity	Effort
4	Showed complete understanding of congruence through the use of congruent shapes	Used irregular or interesting regular shapes to form tessellations	Put in a great deal of effort to create an artistic tessellation piece which has attractive colours and looks pleasant
3	Showed almost complete understanding of congruence through the use of congruent shapes	Used quite interesting regular shapes to form tessellations	Put in very good effort to create a tessellation piece which has some attractive colours and looks pleasant
2	Showed some understanding of congruence through the use of congruent shapes	Used simple geometrical shapes (e.g. square, rectangle, triangle, etc.) to form tessellations	Put in some good effort to create a moderately artistic tessellation piece which is rather plain-looking
1	Showed limited understanding of congruence	Used shapes that could not be properly tessellated	Made some effort in colouring
0	Did not show any understanding of congruence	No tessellations shown	Made little or no effort; poor or no colouring; slipshod and tardy work
Score			

Final Score:

____ / 12

Final Score	10–12	8–9	6–7	4–5	0–3
Grade	A	B	C	D	F

Teacher's Comments (if any):

Performance Task: Scale Drawing

Make a scale drawing of the floor plan of your house using a suitable scale so that it can fit in the space provided below. Label the rooms properly. You should also include some furniture in your floor plan to make it more attractive. The furniture should also be properly labelled and drawn to scale. To know how a floor plan with furniture looks like, you can collect some brochures of HDB or condominium flats.

Scale: _____

Scoring Rubric:

Competency Level	Mathematical Concept	Effort
4	Showed complete understanding of scale drawing and scale	Well-drawn and accurate floor plan with clearly-labelled rooms and furniture
3	Showed almost complete understanding of scale drawing and scale	Made very good effort in drawing the floor plan with furniture but with some minor inaccuracies
2	Showed some understanding of scale drawing and scale	Made some good effort in drawing the floor plan but with some errors or only a few pieces of furniture
1	Showed limited understanding of scale drawing and scale	Made some effort in drawing the floor plan but with a lot of errors and/or no furniture
0	Showed no understanding of scale drawing and scale	Made little or no effort; slipshod and tardy work
Score		

Final Score:

_____ / 8

Final Score	7–8	6	4–5	3	0–2
Grade	A	B	C	D	F

Teacher's Comments (if any):

_____

_____

Chapter 2 Direct and Inverse Proportions

Summary

1. A proportion is a statement expressing the equivalence of two rates or two ratios.

2. If y is directly proportional to x, then
   (a) $\frac{y}{x} = k$ or $y = kx$, where k is a constant and $k \neq 0$;
   (b) the graph of y against x is a straight line that passes through the origin.

3. If y is inversely proportional to x, then $xy = k$ or $\frac{y}{x} = k$, where k is a constant and $k \neq 0$.

Practice Questions

1. 7 litres of petrol cost \$9.45. Find the cost of 23 litres of petrol of the same grade. How many litres can you buy with \$60? Give your answer correct to 3 significant figures.

2. The number of boys to girls in a school is in the ratio 12 : 13. If the difference in the number of boys and girls is 56, how many boys are there in the school?

3. The ratio of female teachers to male teachers in a Junior College is 13 : 5. If there are 48 more female teachers than male teachers, find the total number of teachers in the Junior College.

4. The mass of Aravin to the mass of Bob is in the ratio of 17 : 13. If the difference in their masses is 12 kg, find their combined mass.

5. The ratio of teachers to pupils in a school is 2 : 35. If the total number of teachers and pupils in the school is 1517, find the number of teachers in the school.

6. An alloy is made by mixing the mass of iron, copper and zinc in the ratio 11 : 1 : 2. If the mass of a piece of alloy weighs 882 kg, find the mass of iron in the alloy.

7. The ratio of soft drink cans collected by Ahmad, Betty and Carol is 9 : 7 : 8. Together they collected a total of 1344 soft drink cans. Find the difference between the number of soft drink cans collected by Ahmad and Betty.

8. A coffee shop owner blends three types of coffee, Brazilian, African and Indonesian, in the ratio 8 : 11 : 21. If the mass of Indonesian coffee in the mixture is 63 kg, find the mass of Brazilian coffee in the mixture.
   Find the difference in the mass of the Brazilian and African coffee in 1424 kg of the mixture.

9. 220 marbles are to be divided among Ali, Bala and Charles. The number of marbles received by Ali and Bala are in the ratio 7 : 6 while the number of marbles received by Bala and Charles are in the ratio 4 : 5. Calculate the number of marbles received by each boy.

10. Two identical containers are filled with a mixture of fruit juice and alcohol in the ratio of 5 : 3 and 4 : 7. The contents of the two containers are poured into a big bowl and mixed thoroughly. Find the ratio of fruit juice to alcohol in the big bowl.

11. Given that y is directly proportional to x and that $y = 40$ when $x = 200$, find the value of
(a) y when $x = 15$, **(b)** x when $y = 8$.

12. Given that a is directly proportional to b and that $a = 75$ when $b = 15$, find the value of
(a) a when $b = 37.5$, **(b)** b when $a = 195$.

13. Given that h is directly proportional to l, copy and complete the following table.

h	15	30		75
l		36	72	

14. If s is directly proportional to the square of t and $s = 8$ when $t = 4$, find
(a) s when $t = 3$, **(b)** t when $s = 32$.

15. If n is directly proportional to the cube of m and $n = 27$ when $m = 1\frac{1}{2}$, find
(a) n when $m = 2$, **(b)** m when $n = 125$.

16. The variables x and y are connected by the equation $y = k\sqrt{x+1}$, where k is a constant. Pairs of corresponding values are given in the table below.

x	224	-1	q
y	5	p	$3\frac{1}{3}$

Calculate the values of
(a) k, **(b)** p, **(c)** q.

17. If y is directly proportional to $(4x + 1)$ and $y = 3$ when $x = 2$, find
(a) y when $x = 5$, **(b)** x when $y = 11$.

18. Given that y is directly proportional to $(x + 2)(x + 7)$ and $y = 4$ when $x = 1$, find the value of y when $x = 5$.

19. Given that the square of h is directly proportional to l and that $h = \frac{1}{2}$ when $l = \frac{1}{8}$, find
(a) h when $l = 8$, **(b)** l when $h = 6$.

20. Given that D^3 is directly proportional to L and that $D = 2$ when $L = 6$, find
(a) D when $L = 48$, **(b)** L when $D = \frac{2}{3}$.

21. If y is inversely proportional to the square of x, and $y = 10$ when $x = 10$, find y when $x = 4$.

22. If u is inversely proportional to the square root of v, and $u = 10$ when $v = 9$, find u when $v = 25$.

23. Given that y is inversely proportional to x^2 and that $x = 4$ when $y = 5$, find the value of y when $x = 2$.

24. If y is inversely proportional to $\sqrt{x}$ and $y = 5$ when $x = 16$, find the value of y when $x = 100$.

25. Given that y is inversely proportional to r^2, and $y = 32$ when $r = 1$, find the value of y when $r = 8$.

26. Given that the mass m of a sphere is directly proportional to the cube of its radius r, and $m = 54$ when $r = 3$, find the value of m when $r = 4$.

27. The horizontal force, F newtons, required to push a box of mass m kg along a rough horizontal surface is such that F is directly proportional to m. If $F = 60$ when $m = 250$, find an equation expressing the relationship between F and m.
Hence calculate the value of
(a) F when $m = 300$, **(b)** m when $F = 102$.

28. The interest, $\$I$, on a deposit of $\$P$ for one year at $5\frac{1}{2}\%$ is directly proportional to the deposit. Find the formula relating I and P, and use it to determine I when $P = 1600$.

29. The distance, s metres, through which a heavy object falls from rest is directly proportional to the square of the time, t seconds, taken. Given that $s = 45$ when $t = 3$, find
(a) how far the object will fall from rest in 7 seconds,
(b) how long the object will take to fall 20 metres.

30. The length stretched by a spring is directly proportional to the amount of force applied. A force of 20 kg stretches a certain spring 15 cm. How long will a force of 25 kg stretch it?

31. The distance d travelled by a ball dropped from an airplane is directly proportional to the square of time t. Given that $t = 2$ seconds when $d = 24$ metres,
(a) write down the formula connecting d and t,
(b) find the distance the ball drops in 10 seconds.

32. The mass, w grams, of silver deposited on a metallic surface is directly proportional to the number of hours, t, during which the electric current is being kept steady. If $w = 1.8$ when $t = 0.3$, find the equation connecting w and t. Using this equation, calculate the number of grams of silver that will be deposited in 2.5 hours.

33. The capacity, c, of a set of similar containers is directly proportional to the cube of their heights, h. If a container of height 20 cm holds 2 litres, find
(a) the capacity of a container which is 14 cm high,
(b) the height of a container which has a capacity of 3456 cm^3.

34. The air resistance, R newtons, to the motion of a vehicle is directly proportional to the square root of its speed, v m/s. If the air resistance is 2400 newtons when the speed is 16 m/s, calculate R when $v = 56\frac{1}{4}$.

35. The rate v cm^3/s, at which water flows from a valve at the foot of a tank is directly proportional to the square root of the depth of water, h cm. If the rate is 112 cm^3/s when the depth is 64 cm, calculate v when $h = 30\frac{1}{4}$.

36. The energy E joules stored in an electric string is directly proportional to the square of the extension x cm. Given that when the string is extended by $2\frac{1}{2}$ cm, the energy stored is 175 joules, find

(a) the energy stored when the extension is $4\frac{1}{2}$ cm,

(b) the extension when the stored energy is 252 joules.

37. When a light rod (whose mass may be neglected) carries a load at its mid-point, the sag, S cm, is directly proportional to the cube of its length L m. Given that $S = 1\frac{1}{2}$ when $L = 4\frac{1}{2}$, calculate

(a) the sag when the length is 9 m,

(b) the length when the sag is $\frac{3}{16}$ cm.

38. The safe speed v m/s, at which a train can round a curve of radius r m is directly proportional to the square root of r. If the safe speed for a radius of 121 m is 22 m/s, calculate

(a) the safe speed for a radius of 81 m,

(b) the radius if the safe speed is 28 m/s.

39. The pressure P on a disc immersed in a liquid at a certain depth is directly proportional to the square of the radius R of the disc. Given that the pressure on a disc of radius 2 m is 2880 N/m^2, calculate the pressure on a disc of radius 3.5 m.

40. The heat H produced in a wire is directly proportional to the square of the current I. When a current of 4 amperes flows for 5 minutes, 2880 joules of heat are produced. Find

(a) the heat produced when 5.5 amperes flows for 5 minutes,

(b) the current which flows for 5 minutes and produces 1125 joules of heat.

41. The period P of oscillation of a simple pendulum is directly proportional to the square root of its length l. When the length is 64 cm, the period is 3.2 seconds. Find

(a) the period when the length is 144 cm,

(b) the length when the period is 2 seconds.

42. The diameter of a sphere d is directly proportional to the cube root of its mass w. Given that the mass is 27 kg when the diameter is 21 cm, find the radius when the mass is 512 kg.

43. The pressure P of an enclosed gas, held at a constant temperature, is inversely proportional to the volume V of the gas. The pressure of a certain mass of gas is 500 N/m^2 when the volume at a fixed temperature is 2 m^3. Find the pressure when the volume is 5 m^3.

44. The frequency of radio waves is inversely proportional to their wavelengths. Given that the wavelength is 1.5×10^3 metres when the frequency is 2.0×10^2 kc/s, find
(a) the frequency of radio waves with a wavelength of 480 metres,
(b) the wavelength of radio waves which have a frequency of 960 kc/s.

45. The resistance R of a copper wire of a constant length is inversely proportional to the square of its diameter d. If the resistance of a wire 2.5 mm in diameter is 20 ohms, find the resistance of a wire with diameter 2 mm.

46. The number of days d required to renovate a house is inversely proportional to the number of men available, n. When 6 men are doing the job, the renovation takes 8 days. If it takes 12 days to complete the job, how many men are there?

47. When a shaft is turning at a constant speed, the horsepower that it can transmit is directly proportional to the cube of its diameter. If a 6 cm shaft turning at a constant speed transmit 120 horsepower, what horsepower can a 9 cm shaft turning at the same constant speed transmit?

48. The surface area A of a sphere is directly proportional to the square of its diameter d, i.e. $A = kd^2$.
(a) Can you suggest a value of k?
(b) Given that $A = 38\frac{1}{2}$ when $d = 3\frac{1}{2}$, find the value of k. Is this value of k the same as that you have suggested?
(c) State the relation between A and d in another way.

49. The mass of an object is inversely proportional to the square of the distance from the object to the centre of the earth. A certain astronaut weighs 80 kg at sea level (6500 km from the centre of the earth). How much does the astronaut weigh when orbiting 2.5×10^4 km above the sea level? How far above the earth, to the nearest km, will an astronaut weigh one-half of his or her sea-level mass?

50. When a space satellite orbits the earth, the force F attracting it towards the earth is inversely proportional to the square of its distance R from the centre of the earth. Express F in terms of R and the constant of variation k. Hence calculate
(a) the value of k if $F = 50$ when $R = 32$, (b) the value of R if $F = 512$.

51. The duration, t hours, of an express train travelling from River Dale Town to Queen's Bridge is inversely proportional to the average speed, v km/h. Given that the express train travelling at 80 km/h takes 5 hours to travel from River Dale Town to Queen's Bridge, find the relation between t and v.
On another occasion, the same express train leaves River Dale Town at 09 55 for Queen's Bridge and arrives at 15 15. Use the relation between t and v found above to find the average speed of the express train for the journey.

52. The total cost, $\$c$, of manufacturing n units of biscuit boxes is given by the formula $c = an + b$, where a and b are constants. When 200 units of biscuit boxes are manufactured, the total cost is \$55 000 and when 500 units of biscuit boxes are manufactured, the total cost is \$62 500. Find

(a) the value of a and of b,

(b) the total cost of producing

(i) 420 units, **(ii)** 1250 units of biscuit boxes.

53. An insurance company uses a particular method for determining the annual premium $\$p$ for a life insurance policy. A flat annual fee of \$25 is charged for all policies plus \$2 for each thousand dollars of the amount $\$n$ of the policy. The formula connecting p and n is given by

$$p = 25 + \frac{n}{500}.$$

(a) Calculate the annual premium payable for a \$20 000 policy.

(b) A man pays an annual premium of \$155 for a policy. What is the face value of the policy?

54. Two quantities s and t vary such that $s = at + bt^2$. If $s = 82$ when $t = 2$ and $s = 171$ when $t = 3$, find the value of s when $t = 4$.

55. Two variables x and y are such that $y = ax + \frac{b}{x^2}$. Given that $y = -2$ when $x = 1$ and $y = -11$ when $x = 2$, find the value of y when $x = -2$.

Mind Map

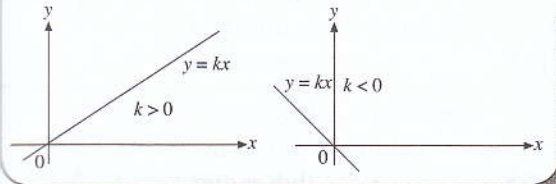
Proportion

- **Direct Proportion**
- **Inverse Proportion**

1. y is directly proportional to x

$\Rightarrow y = kx,\ k \neq 0$

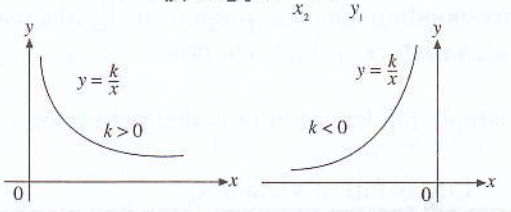
$\frac{y_1}{x_1} = \frac{y_2}{x_2}$ or $\frac{y_1}{y_2} = \frac{x_1}{x_2}$



1. y is inversely proportional to x

$\Rightarrow y = \frac{k}{x},\ k \neq 0$

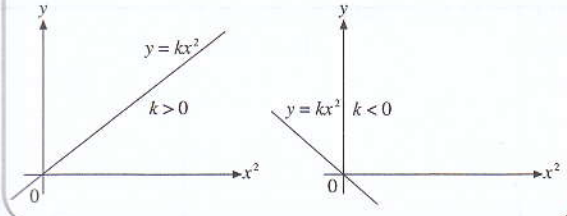
$x_1y_1 = x_2y_2$ or $\frac{x_1}{x_2} = \frac{y_2}{y_1}$



2. y is directly proportional to x^2

$\Rightarrow y = kx^2,\ k \neq 0$

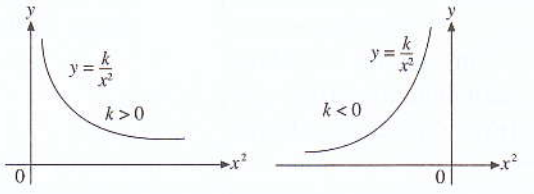
$\frac{y_1}{(x_1)^2} = \frac{y_2}{(x_2)^2}$ or $\frac{y_1}{y_2} = \frac{(x_1)^2}{(x_2)^2}$



2. y is inversely proportional to x^2

$\Rightarrow y = \frac{k}{x^2},\ k \neq 0$

$(x_1)^2y_1 = (x_2)^2y_2$ or $\frac{(x_1)^2}{(x_2)^2} = \frac{y_2}{y_1}$



Alternative ASSESSMENT

Rap: Direct and Inverse Proportions in Our Lives

J. B. B. Token is recruiting members for his Rap & Rapper's Society. To apply for membership, you need to compose a rap with at least 8 lines to demonstrate that you have the flair to be a rapper. The theme is ***Direct and Inverse Proportions in Our Lives***. You have to include key and relevant mathematical terms for the topic in your rap. Be prepared to present your rap in front of an audience.

Examples of key terms that you can use in your rap are: directly proportional, inversely proportional, corresponding, increase proportionally, decrease proportionally, doubled, tripled, halved, constant, rate, ratio, variables, values, reciprocal.

A sample rap has been provided below for you:

Life is full of variables,
With matters sometimes blown out of proportion, blown out of proportion;
Variation is a way of change,
Change is a constant, a constant;
Just like the chance of the bread falling, falling,
With the buttered side down is directly proportional, proportional,
To the cost of the carpet, carpet.

Scoring Rubric:

Competency Level	Creativity	Mathematical Communication	Effort
4	Composed an original and fluent rap which was appealing	Used a variety of key mathematical terms appropriately in the rap	Put in a great deal of effort to compose the rap and/or to present the rap before an audience in a lively and exciting manner
3	Composed an original and relatively fluent rap with some room for improvement	Used a variety of key mathematical terms but some were not used appropriately in the rap	Put in very good effort to compose the rap and/or to present the rap before an audience
2	Composed a rap which was quite fluent but rather dull	Used some key mathematical terms but limited in variety and not very appropriately	Put in some good effort to compose the rap and/or to present the rap before an audience
1	Composed a rap with some flow but not fluent	Used few key mathematical terms in the rap and/or not appropriately	Put in some effort to compose the rap and/or to present the rap before an audience
0	Composed a rap with no flow at all	Did not use any key mathematical terms	Put in little or no effort to compose the rap and/or to present the rap before an audience; slipshod and tardy work
Score			

Final Score:

____ / 12

Final Score	10–12	8–9	6–7	4–5	0–3
Grade	A	B	C	D	F

Teacher's Comments (if any):

Chapter 3 Expansion and Factorisation of Algebraic Expressions

Summary

1. Algebraic identities:
   (a) $(a + b)^2 = a^2 + 2ab + b^2$
   (b) $(a - b)^2 = a^2 - 2ab + b^2$
   (c) $(a + b)(a - b) = a^2 - b^2$

2. Factorisation of algebraic expressions can be done by
   (a) identifying and taking out all the common factors from every term in the given expressions;
   (b) grouping terms in such a way that the new groups obtained have some common factors;
   (c) using the 'cross' method for quadratic expressions.

3. If two factors P and Q are such that $P \times Q = 0$, then either $P = 0$, or $Q = 0$, or both P and Q are equal to 0. This principal is used to solve quadratic equations.

Practice Questions

1. Expand each of the following.
   (a) $3(2x + 7y)$ (b) $4(3h - 5k)$ (c) $-4(2a + 3b)$ (d) $-6(-3x + 7y)$
   (e) $-5(-2h - 9k)$ (f) $7(-5h - 7k)$ (g) $-8(4p - 3q)$ (h) $9(-2h + 3k)$

2. Expand each of the following.
   (a) $5x(2x + 3y)$ (b) $-6x(y - 4x)$ (c) $-3m(-2m - n)$ (d) $4h(-2k - 3h)$
   (e) $9a(-4a + 7b)$ (f) $-4y(2x + 5y)$ (g) $-7x(-3x + 4y)$ (h) $8p(5p - 2q)$

3. Expand and simplify each of the following.
   (a) $3(x + 2) + 4(2x + 3)$ (b) $6(p + 3) - 5(p - 4)$
   (c) $8(5 - 4x) - 7(7 - 5x)$ (d) $11(5x - 7) + 9(2 - 3x)$
   (e) $13(5x + 7) - 6(3x - 5)$ (f) $9(3p - 2) - 5(2 + p)$
   (g) $8(5a - 4) + 3(2 - 4a)$ (h) $7(12 - 5x) - 3(9 - 7x)$

4. Expand and simplify each of the following.
   (a) $2x(3x + 4) + x(5x - 2)$ (b) $5x(x + 3) - 4x(5 - x)$
   (c) $4x(3x - y) - 2x(5y - x)$ (d) $5p(2p + 5q) - 3p(2q - 7p)$
   (e) $2a(4b - 3a) - 5a(2b - 5a)$ (f) $7x(2x + 3y) - 3x(3x - 4y)$
   (g) $5x(-2x - 3y) + 2x(-x + 3y)$ (h) $4p(-3p + q) - 2p(-5q + p)$

5. Expand each of the following.

(a) $(x + 5)(x + 7)$ (b) $(x + 11)(x - 7)$ (c) $(7 - 2x)(4 + x)$

(d) $(x^2 + 3)(2x - 4)$ (e) $(x^2 - 4)(2x + 3)$ (f) $(2x - 3y)(x - 2y)$

(g) $(4x - 5)(3x + 4)$ (h) $(4x - 3y(2x + 7y)$ (i) $(4x + 5y)(5x + 7y)$

(j) $(4x + 3)(4x - 3)$ (k) $(2x - 3b)(2x - 5c)$ (l) $(ab - 5)(ab + 8)$

(m) $(x - 1)(x + 2)(x - 3)$ (n) $(x - 2)(x + 2)(x^2 + 4)$ (o) $(x^2 - y)(x^2 + y)(x^4 + y^2)$

6. Expand each of the following.

(a) $(3x + y)^2$ (b) $(6x + 5y)^2$ (c) $\left(x + \frac{2}{x}\right)^2$

(d) $\left(x + \frac{y}{3}\right)^2$ (e) $\left(3x + \frac{1}{4}y\right)^2$ (f) $(7x - y)^2$

(g) $(5x - 9y)^2$ (h) $(xy + 2)^2$ (i) $(x^2 + 3)^2$

(j) $(x^2y + z)^2$ (k) $(abc - x)^2$ (l) $(x^3 + 4)^2$

(m) $\left(\frac{a}{b} + \frac{1}{c}\right)^2$ (n) $\left(\frac{2}{x} + \frac{3}{y}\right)^2$ (o) $\left(\frac{a}{bc} - 3\right)^2$

(p) $\left(\frac{a}{b} + \frac{b}{a}\right)^2$ (q) $\left(\frac{x^2}{y} - \frac{y}{x}\right)^2$ (r) $\left(\frac{a}{b} + 3b\right)^2$

7. Expand each of the following.

(a) $(3 - a)(9 + 3a + a^2)$ (b) $(x + y)(x^2 - xy + y^2)$ (c) $(2a + b)(3a - 4b + c)$

(d) $(2x + 1)(x^2 - 3x - 4)$ (e) $(x^2 - 4)(x^2 - 2x + 1)$ (f) $(a + 2)(3a^2 - 5a + 6)$

(g) $(5 - 2a)(2 - 3a - a^2)$ (h) $(7 - a)(5a^2 - 2a + 1)$ (i) $(p - 2q)(2p + 3q - 1)$

(j) $(x + 1)(x^3 - x^2 + x - 1)$ (k) $(x - y)(x^2 + xy + y^2)$ (l) $(a - 1)(a^3 - 3a^2 + 3a - 1)$

8. Simplify each of the following.

(a) $(3x + y)(x - 2y) - 2(x - y)^2$ (b) $(x - y)^2 + 2y(x + y) - (x^2 - y^2)$

(c) $(7x + 1)(x - 5) - 3(4 - 2x - x^2)$ (d) $(3x - 8)(x + 1) - (2x - 1)(5 - x)$

(e) $(a + 1)(a - 3) + (2a - 3)(5 - 7a)$ (f) $(2x + 3)(x - 7) - (x + 4)(x^2 - 1)$

(g) $(x + 3y)(x - 3y) - 2(x + 2y)(x - y)$ (h) $(3x^2 + y)(2x - y) - (2x + y)(3x^2 - y)$

9. Factorise each of the following where possible.

(a) $24x + 16$ (b) $2ab + 4abc$ (c) $abc - a^2bc^3$

(d) $2x + kx$ (e) $5ab - 8cd$ (f) $2a^2b^3c - 8ab^2c^3$

(g) $p^2q - 2pq^2 + 4p^2q^2$ (h) $a^2b - ab^2 + a^2b^2$ (i) $2x - 4x^2 + 8xy^2$

(j) $5a^2x - 3a^3x^2 + 6a^2x^2$ (k) $6a^2 + 8a^3 - 10a^5$ (l) $12x^3y - 9x^2y^2 + 6xy^3$

*10. Factorise each of the following.

(a) $a(b - c) + bc - a^2$ (b) $x^2 + xy + 3yz + 3xz$

(c) $x^2z - 4y - x^2y + 4z$ (d) $8ab - 6bc + 15cd - 20ad$

(e) $2ax - 4ay + 3bx - 6by$ (f) $x^3 + xy - 3x^2y - 3y^2$

(g) $a^2 - 1 + ab + b$ (h) $3xy + 6y - 5x - 10$

(i) $a^2 - 3bc - ab + 3ac$ (j) $x^2y - 3y - 6 + 2x^2$

(k) $x - 4x^2 - 4 + x^3$ (l) $x^3 - 12 + 4x - 3x^2$

(m) $x - y - x^2 + y^2$ (n) $4x^2 - y^2 + 6x + 3y$

(o) $(x + 5)(x - 1) + 5a + ax$ (p) $6y + 3x^3y + x^4 + 2x$

(q) $a^2x - 12by - 3xb + 4a^2y$ (r) $5x^2 - 4yz + 5xz - 4xy$

11. Factorise each of the following.

(a) $\frac{1}{4}x^2 - y^2$ **(b)** $9a^2 - x^2$ **(c)** $4a^2 + 4ab + b^2$

(d) $9a^2 - 6ab + b^2$ **(e)** $(a + 2b)^2 - a^2$ **(f)** $x^4 + 8x^2 + 16$

(g) $a^2b^2 - 10ab + 25$ **(h)** $25x^2 - 20x + 4$ **(i)** $\frac{1}{4}x^2 - \frac{1}{4}xy + \frac{1}{16}y^2$

(j) $9a^2 + 12a + 4$ **(k)** $16a^2 + 40ab + 25b^2$ **(l)** $49a^2 - 28ab + 4b^2$

(m) $4x^2 - 81$ **(n)** $81ab^2 - 4ac^2$ **(o)** $x^4 - 81y^4$

12. Factorise each of the following.

(a) $x^2 - 2x - 35$ **(b)** $2x^2 + 8x - 42$ **(c)** $3x^2 - 5x - 2$

(d) $2x^2 - 5x - 3$ **(e)** $x^2 + 20x + 75$ **(f)** $x^2 - 11x + 28$

(g) $x^2 + 4x - 77$ **(h)** $x^2 + 3x - 154$ **(i)** $x^2 - 21x + 68$

(j) $x^2 - 10x - 171$ **(k)** $12x^2 - 31x - 15$ **(l)** $15x^2 + 2x - 1$

(m) $3x^2 - 36x + 108$ **(n)** $3x^2 + 11x - 20$ **(o)** $3x^2 - x - 10$

13. Factorise each of the following.

(a) $4x^3 - 49x$ **(b)** $5x^2 - 20$ **(c)** $27a^3 - 48a$

(d) $9 - (a - b)^2$ **(e)** $(x - 3)^2 - 16y^2$ **(f)** $18x^3 - 8xy^2$

(g) $x^4 - 25x^2$ **(h)** $49 - x^2$ **(i)** $4x^2y - 8xy^2$

(j) $4x^2 - (p - 2)^2$ **(k)** $3x^2 - 12y^2$ **(l)** $(3x - 2y)^2 - (2x - 3y)^2$

(m) $2x^3 + 3x^2 - 2x$ **(n)** $6x^2 - 7xy - 10y^2$ **(o)** $(3x - y)^2 - x^2$

(p) $9x^2 - (3x - 2y)^2$ **(q)** $(t^2 - 1)^2 - 9$ **(r)** $9x^2 - 4(x - 2y)^2$

(s) $6x^3 - x^2y - 35xy^2$ **(t)** $81x^5y^3 - 121x^3y^5$

14. Use algebraic rules to evaluate each of the following (calculators not allowed).

(a) 99×101 **(b)** 8001^2 **(c)** 603×597

(d) $201^2 - 99^2$ **(e)** $462^2 - 452^2$ **(f)** $823^2 - 177^2$

(g) $1.013^2 - 0.013^2$ **(h)** $201^2 - 402 + 1$ **(i)** $65^2 + 650 + 25$

(j) $41^2 + 738 + 81$ **(k)** $92^2 - 368 + 4$ **(l)** $15\,316^2 - 14\,316^2$

15. Factorise $3x^2 + 26x + 51$. Hence or otherwise find two factors of 32 651.

16. If $(a + b)^2 = 73$ and $ab = 6.5$, calculate the value of $a^2 + b^2$.

17. Factorise $a^2 - b^2$. Hence evaluate the value of $2030^2 - 2029^2 + 2028^2 - 2027^2$.

18. If $x^2 + y^2 = 43$ and $4xy = 48$, calculate the value of

(a) $(x + y)^2$, **(b)** $(2x - 2y)^2$.

19. If $x^2 - y^2 = 6$ and $x - y = 2$, find the value of $(x + y)^2$.

20. Factorise $4x^2 + 13x + 3$ and use your result to find the prime factors of 41 303.

21. Given that $x + 2y = -2$ and $x - 2y = 18$, find the value of

(a) $x^2 - 4y^2$, **(b)** $x^2 + 4y^2$.

22. Evaluate $10^2 - 9^2 + 8^2 - 7^2 + 6^2 - 5^2 + 4^2 - 3^2 + 2^2 - 1^2$ by using algebraic method.

23. Evaluate the value of $2008^2 - 2007^2 + 2006^2 - 2005^2 + 2004^2 - 2003^2$ by using algebraic method.

24. Solve the following equations.

(a) $3x(x - 5) = 0$
(b) $5x(3x - 2) = 0$
(c) $7y(9y + 4) = 0$
(d) $8p(7 - 5p) = 0$
(e) $(x - 5)(2x - 7) = 0$
(f) $(5x + 9)(8 - 3x) = 0$
(g) $(7p - 5)(2 - 9q) = 0$
(h) $(6 - 5h)(15 + 11k) = 0$

25. Solve the following equations.

(a) $2x^2 + 5x = 0$
(b) $7x - 8x^2 = 0$
(c) $2x^2 = 7x$
(d) $(x + 2)^2 = 9$
(e) $(x - 3)^2 = 25$
(f) $(x + 3)^2 = 16$
(g) $(2x + 5)^2 = 7(2x + 5)$
(h) $(x - 2)^2 = 9(x - 2)$
(i) $5x^2 - 5 = 0$
(j) $x^2 - 4 = 12$

26. Solve the following equations.

(a) $2x^2 + 7x - 4 = 0$
(b) $5x^2 + 17x + 6 = 0$
(c) $2x^2 - 3x - 14 = 0$
(d) $12x^2 - 7x - 12 = 0$
(e) $12x^2 - x = 6$
(f) $6x^2 = x + 15$
(g) $9x^2 - 3x = 20$
(h) $8x^2 - 22x = 63$
(i) $8x^2 + 10x - 3 = 0$
(j) $10 - 19x - 15x^2 = 0$
(k) $9x^2 - 6x - 120 = 0$
(l) $3 - 4x - 7x^2 = 0$
(m) $x^2 = 10x + 24$
(n) $8 - 18x - 5x^2 = 0$
(o) $x^2 + 4 = 8x - 8$
(p) $x(2x + 5) = 3$
(q) $2x^3 - 5x^2 - 3x = 0$
(r) $6x^3 - x^2 = x$
(s) $(6x + 5)(x - 1) = -3$
(t) $6(x - 1)^2 = 16 - 8x$

*27. Solve the following equations.

(a) $(2x - 1)^2 = (4x - 5)(x + 3)$
(b) $4(x^2 - 2x - 3) = 5(x - 3)$
(c) $(3x - 2)(2x + 1) = (6x + 5)(x - 2) + 7$
(d) $6x^2 + x - 3 = 9$
(e) $\frac{1}{x} - \frac{1}{x+3} = \frac{1}{36}$
(f) $\frac{x-3}{4} + \frac{1}{x-1} = \frac{1}{2}$
(g) $9x + \frac{4}{x-1} = 46$
(h) $\frac{x-3}{2} = \frac{4}{x+4}$
(i) $\frac{3}{x} = \frac{2x-7}{5}$
(j) $\frac{2}{x+2} - \frac{3-x}{2} = 0$
(k) $\frac{3x-1}{4} - \frac{1}{x} = \frac{3(2x+1)}{8}$
(l) $\frac{3x-1}{x-1} - 1 = \frac{2x+8}{x+1}$

28. Given that $x = 3$ is one solution of the equation $2x^2 + px = 15$, calculate the value of p and hence find the other solution.

29. Find two consecutive positive odd numbers which are such that the square of their sum exceeds the sum of their squares by 126.

30. The area of a rectangular field is 450 m^2 and the difference between the lengths of the two adjacent sides is 7 m. Find the length of the shorter side and the perimeter of the rectangle.

31. The product of two numbers is 154. If the difference between the two numbers is 3, find the numbers.

32. The length and breadth of a rectangle are $(4x + 7)$ cm and $(5x - 4)$ cm respectively. If the area of the rectangle is 209 cm^2, find
(a) the value of x,
(b) the perimeter of the rectangle.

33. The sum of a number and twice its square is 36. Find the number.

34. The sum of the squares of two consecutive even integers is 340. Find the two numbers.

35. The length of the parallel sides of a trapezium are $(x + 3)$ cm and $(x + 9)$ cm and its height is $(3x - 4)$ cm. If its area is 80 cm^2, find the value of x.

36. The sum of the squares of three consecutive positive numbers is 245. Find the largest number.

37. Fatimah is 5 years older than Dollah. If the product of their ages is 234, how old is Dollah?

38. A car travels a 750-km journey at an average speed of x km/h. If it had increased its speed by 18 km/h, the journey would have been 125 minutes shorter. Form an equation in x and show that it reduces to $x^2 + 18x = 6480$. Solve this equation to find the value of x. Hence, find the time taken when the car travels at x km/h.

39. 1 kg of prawn was sold at $\$x$. During a lean season, the cost increased by \$3 a kg. As a result of this increase, a man found that he got 5 kg less for \$300. Form an equation in x and hence solve it.

40. The lengths of a right-angled triangle are $(x + 2)$ cm, $(5x - 1)$ cm and $5x$ cm. Form an equation in x and show that it reduces to $x^2 - 6x + 5 = 0$. Solve this equation to find the two possible values of x. Hence find the area and perimeter of the triangle for each value of x.

41. The sides of 2 square fields are in the ratio of 3 : 5. The area of the larger field is 576 m^2 greater than the area of the smaller field. Find the area of this smaller field.

42. The length of a rectangle exceeds its breadth by 8 cm. If the length was halved and the breadth increased by 6 cm, the area would be decreased by 36 cm^2. Find the length and perimeter of the original rectangle.

43. Show that the sum of any three consecutive even numbers is divisible by 6.

44. The sum S of the first n integers is given by the formula $S = \frac{1}{2}n(n + 1)$. How many integers must be taken to have a sum of 325?

45. The sides of rectangle A are $5x$ cm and $(4x + 2)$ cm. The sides of rectangle B are $(6x + 3)$ cm and $(3x + 1)$ cm. If the area of A is equal to the area of B, find x. Which rectangle has a longer perimeter?

46. $5x$ articles cost $(8x + 5)$ dollars while $2x$ similar articles cost $(3x + 4)$ dollars. Find x.

47. The difference between two positive integers is 4 and the difference between their reciprocals is $\frac{1}{24}$. Find the integers.

48. When x^2 is divided by $(x - 3)$, the quotient is 12 and the remainder is 1. Find the possible values of x.

49. The sides of a rectangle are of lengths $(2x + 1)$ cm and $(3x + 1)$ cm. The area of the rectangle is 117 cm^2. Find x and the perimeter of the rectangle.

50. Factorise $3x^2 + 48x + 189$ completely. Hence or otherwise, express 969 as a product of three prime numbers.

51. Find two positive whole numbers which differ by 5 and where the sum of their squares is 193.

52. Show that the sum of any four consecutive odd numbers is divisible by 8.

Exploratory Worksheet: Factorisation of Quadratic Expressions

- 12 can be written as a product of two factors, e.g. $12 = 1 \times 12$ (trivial), $12 = 2 \times 6$ or $12 = 3 \times 4$.
- Similarly, how do you factorise $x^2 + 3x$ or $x^2 + 5x + 6$ into two non-trivial factors $x + p$ and $x + q$ where p and q are integers?

Section A: Introduction to Algebra Tiles [1]

Tile	Dimensions of Tile	Area of Tile
Large square 	$x \times x$	
Rectangle 	$x \times 1$	
Smaller square 	1×1	

- **Note:** $x \neq 5$ (x can be ***any*** value)

Section B: Factorisation of $x^2 + bx$

- Try to arrange the algebra tiles (represented by the following equations) in the form of a rectangle. **[1]**

Expression	Pictorial Arrangement (Can you form a rectangle?)	Total Area of the Tiles (in terms of length & breadth of rectangle)
$x^2 + x$		
$x^2 + 2x$		
$x^2 + 3x$		

1. When can you factorise $x^2 + bx$? Explain in terms of the algebra tiles. **[1]**

_____

_____

2. How are the factors of $x^2 + bx$ related to the dimensions of the rectangle? **[1]**

_____

_____

Section C: Factorisation of $x^2 + bx + c$

- If you **can't form a rectangle** with the algebra tiles, just write **N.A.** (Not Applicable).
- **Note:** A square is a special rectangle.
- **Note**: c is **constant** for each table; only b varies.

Table C1. Factorisation of $x^2 + bx + 2$ [2]

Expression	Pictorial Arrangement (Can you form a rectangle?)	Total Area of the Tiles (in terms of length & breadth of rectangle)
$x^2 + x + 2$		
$x^2 + 2x + 2$		
$x^2 + 3x + 2$		
$x^2 + 4x + 2$		

Table C2. Factorisation of $x^2 + bx + 6$ [2]

Expression	Pictorial Arrangement (Can you form a rectangle?)	Total Area of the Tiles (in terms of length & breadth of rectangle)
$x^2 + x + 6$		
$x^2 + 2x + 6$		
$x^2 + 3x + 6$		
$x^2 + 4x + 6$		
$x^2 + 5x + 6$		
$x^2 + 6x + 6$		
$x^2 + 7x + 6$		

[2]

Section D: Findings

3. What do you notice about the arrangement of the 6 small square tiles when $x^2 + bx + 6$ can be factorised? **[1]**

4. What do you notice about the values of b when $x^2 + bx + 6$ can be factorised? **[1]**
[**Hint:** Look at the 6 small square tiles.]

5. If $x^2 + bx + 6$ can be factorised in the form $(x + p)(x + q)$, how are p and q related to 6? How are p and q related to b? **[1]**

6. If $x^2 + bx + c$ can be factorised in the form $(x + p)(x + q)$, how are p and q related to c? How are p and q related to b? **[1]**

7. Confirm your observation in Q6 by stating all the values of b for which $x^2 + bx + 12$ can be factorised. **[1]**

8. Use algebra tiles to illustrate Q7. Record your observations below. [2]

Expression	Pictorial Representation	Total Area of the Tiles (in terms of length & breadth of rectangle)

Section E: Conclusion

9. What do you mean when you say $x^2 + bx + c$ can be factorised in the form $(x + p)(x + q)$? Think in terms of algebra tiles. **[1]**

10. When $x^2 + bx + c$ can be factorised in the form $(x + p)(x + q)$, what is the relationship between p, q and c? What is the relationship between p, q and b? **[2]**

11. How do the relationships in Q10 help you to actually factorise $x^2 + bx + c$ without using algebra tiles? Use $x^2 + 11x + 24$ as an example. **[2]**

Note: This method works only for $x^2 + bx + c$ where b and c are **both positive**.

Section F: Enrichment

12. Can all quadratic expressions be factorised? Explain with examples or non-examples.

[Bonus 2 marks]

Final Score: ____ / 20

Final Score	16–20	13–15	10–12	7–9	0–6
Grade	A	B	C	D	F

Teacher's Comments (if any):

Term I Revision Test

Time: $1\frac{1}{2}$ h

1. **(a)** Given that a hawker can cook 12 pratas in 8 minutes, how many minutes will he take to cook 50 pratas? **[2]**

(b) It takes 8 men working 9 hours each to erect 12 tents. How long will 6 men take to erect 32 tents? **[2]**

2. A map is drawn to a scale of 1 : 75 000.

(a) Calculate the distance between two places on the map if they are 15 km apart.

(b) The length of the railway track on the map is 46 cm. Find its actual length in km.

(c) A park on the map has an area of 8 cm^2. Calculate its actual area in km^2.

(d) A forest reserve on the map occupies an area of 3 cm^2. Find the area of the same forest reserve drawn on another map whose scale is 1 : 25 000. **[6]**

3. Simplify

(a) $(x + 3)(x^2 + x + 2)$,

(b) $(3p + 2q)(3p - 2q) - (p + q)^2$,

(c) $\dfrac{4b - 1}{a^2 + 3a} \times \dfrac{a + 3}{4b^2 + 11b - 3}$. **[6]**

4. Factorise the following.

(a) $2x^4 - 32y^2z^2$

(b) $64m^2n^2 - 16mn + 1$

(c) $a^2 - 4b^2 + 3(a - 2b)$ **[6]**

5. **(a)** Two cylindrical water jars are similar. The base radius and height of the smaller jar are 4 cm and 12 cm respectively while that of the larger jar are 6 cm and h cm respectively. Calculate h.

(b) A map is drawn to a scale of 1 : 40 000. A piece of land on the map has an area of 8 cm^2. Calculate its actual area, giving your answer in m^2. **[4]**

6. A shopkeeper bought a certain number of articles for \$560.

(a) Given that each article costs x dollars, write down an expression for the number of articles he bought.

(b) When the price per article was increased by a dollar, he found that he obtained 10 articles fewer for the same amount of money. Form an equation in x and show that it reduces to $x^2 + x - 56 = 0$.

(c) Calculate the original price of each article and the number of articles he bought. **[6]**

7. **(a)** Solve the equation $\dfrac{x}{3} = \dfrac{3x - 5}{2} + 6$.

(b) Solve the equation $(2x + 5)(8x - 1) = (4x + 3)(4x - 3)$. **[4]**

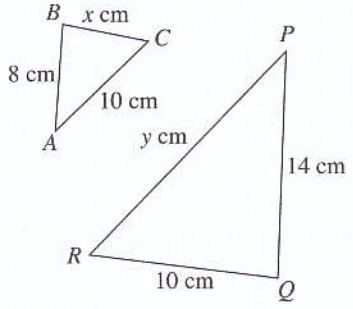
8. A map is drawn to a scale of 1 : 25 000.

(a) Two villages are 4.5 km apart. Calculate, in cm, their distance apart on the map.

(b) A housing estate drawn on the map occupies an area of 40 cm^2. Calculate the actual area of the housing estate, giving your answer in km^2.

If the same housing estate is now drawn on another map whose scale is 1 : 50 000, find its area on the map. **[4]**

9. In the figure, $\triangle ABC$ is similar to $\triangle PQR$, AB = 8 cm, BC = x cm, AC = 10 cm, PQ = 14 cm, QR = 10 cm and PR = y cm. Calculate the value of x and of y. **[4]**

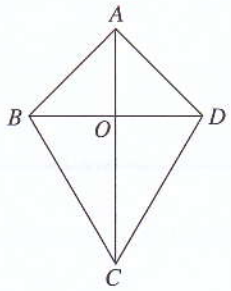


10. **(a)** Given that y is directly proportional to the square of $(x + 3)$ and that $y = 36$ when $x = 0$, calculate the value of y when $x = 2$. **[2]**

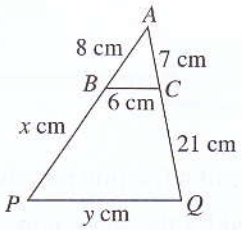
(b) If H is inversely proportional to $(2x - 3)^3$ and that $H = -5$ when $x = 1$, calculate the value of H when $x = 2.5$ and the value of x when $H = \dfrac{5}{27}$. **[4]**

11. A map is drawn to a scale of 4 cm to 5 km.
(a) Calculate the distance between two towns on the map if their actual distance apart is 40 km.
(b) A rubber plantation is represented by an area of 12 cm^2 on the map. Calculate the actual area of the plantation, giving your answer in hectares. [3]

12. The figure below shows a kite $ABCD$ whose diagonals intersect at O. Name a triangle that is congruent to
(a) $\triangle ABO$, (b) $\triangle COD$,
(c) $\triangle ABC$. [3]



13. Given that $\triangle ABC$ is similar to $\triangle APQ$, $AB = 8$ cm, $BC = 6$ cm, $AC = 7$ cm, $CQ = 21$ cm, $BP = x$ cm and $PQ = y$ cm, calculate the value of x and of y. [4]



Chapter 4 Algebraic Manipulation and Formulae

Summary

1. The value of a fraction remains unchanged if both its numerator and its denominator are multiplied or divided by the same non-zero number or expression i.e. $\frac{a}{b} = \frac{a \times c}{b \times c}$ and $\frac{a}{b} = \frac{a \div c}{b \div c}$.

2. Generally, the algebraic method for solving a problem consists of the following steps:
   (1) Let the unknown be denoted by a variable.
   (2) Form an equation involving the variable.
   (3) Solve the equation.
   (4) Check the solution.

Practice Questions

1. Simplify each of the following.

(a) $\frac{45x^2y}{3x}$ (b) $\frac{35x^7y^3}{7xy^4}$ (c) $\frac{64ab^3c^4}{24a^3bc^2}$ (d) $\frac{8x^3yz^4}{(2xyz)^4}$

(e) $\frac{(-3x)^2y^3z}{27xyz^4}$ (f) $\frac{9x^3y^4z}{(3xy^2z)^3}$ (g) $\frac{8xy^2z^3}{(4xyz)^2}$ (h) $\frac{(4x^2)^2y^3z}{8xyz^4}$

(i) $\frac{(7x^2y)^2z^4}{21yz}$ (j) $\frac{(2abc^2)^4}{8a^2b^3}$ (k) $\frac{(-3x^2y^4)^3}{9x^2y^5}$ (l) $\frac{(-4ab^3c^5)^3}{16a^4b^5c}$

(m) $\frac{(5x^3y^4)^3}{25xy^3}$ (n) $\frac{(-9a^4bc)^3}{(27abz)^2}$ (o) $\frac{(-4ab^3c)^3}{-16a^4bc^5}$ (p) $\frac{5a^3b(x+y)}{10a(x+y)^2}$

(q) $\frac{15x^3(a-b)^4}{35xy(a-b)^2}$ (r) $\frac{5x^3y(a+b)}{15xy^2(a-b)}$ *(s) $\frac{15a^n}{25a^{n+3}}$ *(t) $\frac{16a^nb^{n+3}}{48a^{n-1}b^n}$

*(u) $\frac{6a^{n+5}b^{n-2}}{16a^4b^{n-4}}$ *(v) $\frac{49a^{n-1}b^n}{7a^2b^3}$

2. Simplify each of the following where possible.

(a) $\frac{4hk-8h^2}{6h^2}$ (b) $\frac{a+b}{a-cd}$ (c) $\frac{x^2+xy}{xy+y^2}$

(d) $\frac{a^2-ab}{b^2-ab}$ (e) $\frac{a^2+b^2}{a^2-b^2}$ (f) $\frac{4a+8b}{6a+12b}$

(g) $\frac{a^2-b^2}{(a-b)^2}$ (h) $\frac{x^2-4x}{x^2-16}$ (i) $\frac{ab-b^2}{(a-b)^2}$

(j) $\frac{8a^2-16ab}{5a-10b}$ (k) $\frac{a^2-4a}{a^2-4}$ (l) $\frac{4x^2-y^2}{12x^2-4xy-y^2}$

(m) $\dfrac{x^2+x-6}{x^2-9x+14}$ (n) $\dfrac{5x-15}{3x^2-13x+12}$ (o) $\dfrac{6a^2-13a-5}{6a^2+17a+5}$

(p) $\dfrac{(a+b)^2-c^2}{(b+c)^2-a^2}$ (q) $\dfrac{6-11a+4a^2}{14-a-3a^2}$ (r) $\dfrac{(x+3y)^2-4y^2}{x^2-25y^2}$

(s) $\dfrac{(2x-3y)^2}{6x^2-9xy}$ *(t) $\dfrac{2ac+bc-2ad-bd}{cx-3cy-dx+3dy}$ *(u) $\dfrac{x^2-2xz+xy-2yz}{x^2+xy-xz-yz}$

*(v) $\dfrac{6xz+3yz}{6x^2-2xz+3xy-yz}$ (w) $\dfrac{(3x+y)^2-4z^2}{15x^2+5xy+10xz}$

3. Simplify each of the following.

(a) $\dfrac{16xy^3}{15abc^2}\times\dfrac{25a^3bc}{8x^2yz}$ (b) $\dfrac{4a^2b}{3bc}\times\dfrac{27b^2c^3}{16a^4}$ (c) $\dfrac{5x^2y^4}{3yz^4}\times\dfrac{9y^2}{10x^3}$

(d) $\dfrac{16a^2b^4}{7c^2b}\times\dfrac{21b^4c^3}{24a^3b^3}$ (e) $\dfrac{6xy^2}{7z}\times\dfrac{56x^3}{48yz}$ (f) $\dfrac{9a^3x^2}{4by^2}\times\dfrac{5b^2y^4}{12a^2y}$

(g) $\dfrac{2a^2b}{3c}\div\dfrac{3abc}{8c^3}$ (h) $\dfrac{14a^3b}{6xy}\div\dfrac{21abc}{12x^2y^3}$ (i) $\dfrac{25x^3y}{49xz}\div\dfrac{15xy^2}{21x^3z^2}$

(j) $\dfrac{3x^2y}{8xy^3}\div\dfrac{21xz^4}{49xyz^2}$ (k) $\dfrac{81a^3x^3}{16bxy}\div\dfrac{63ax^2}{24b^2y^3}$ (l) $\dfrac{18x^4y^3}{14x^2y}\div\dfrac{27xy^5}{21yz^2}$

4. Simplify each of the following.

(a) $\dfrac{2a}{b}\times\dfrac{3c}{4a}\times\dfrac{8a}{9c}$ (b) $\dfrac{3p^2}{qr}\times\dfrac{6q^2}{21rq}\times\dfrac{28r^2}{3pq}$

(c) $\dfrac{2}{h^2}\times\dfrac{1}{k^3}\div\dfrac{2h}{3k}$ (d) $\dfrac{3x^3y^3}{8z^4}\times\dfrac{6y^2z^3}{5x^5}\div\dfrac{9y^2}{10az}$

(e) $\dfrac{2x^2y^3}{7az^3}\div\dfrac{4x^2z}{21a^2z}\times\dfrac{3a}{8xy}$ (f) $\dfrac{4m^2n^4}{36m}\times\dfrac{24m}{8m^2n^3}\div\dfrac{16a}{6ab^2}$

*(g) $\dfrac{16a^3b^4}{7xy^4}\div\dfrac{4ab^2}{21xy^3}\times\dfrac{27a^{n+1}}{9a^{n-2}}$ (h) $\dfrac{4pq}{8r}\div\left(\dfrac{2r}{3s}\div\dfrac{8x}{9y}\right)$

(i) $\dfrac{9h}{21k}\div\left(\dfrac{3l}{4m}\times\dfrac{16mh}{9g}\right)$ (j) $\dfrac{3a}{4b}\div\left(\dfrac{7a^2}{15c}\div\dfrac{3b^2}{10c^2}\right)$

(k) $\dfrac{x^5-x^4}{ax-a}\div\dfrac{ax^2}{ax-x}$ (l) $\left(\dfrac{1}{x^3}-\dfrac{1}{x}\right)\div\left(\dfrac{1}{x^2}-\dfrac{1}{x}\right)$

5. Find the H.C.F. of each of the following.

(a) $2a$, $4b$, $8ab$ (b) ab, ac, abc (c) $3a$, $6ab$, $9a^2b$

(d) $5x^2y$, $10xy^2$, $30x^2y^3$ (e) $2mn$, $6m^2n$, $8mn^2$ (f) $11xy$, $55x^2y$, $121x^3y^2$

6. Find the L.C.M. of each of the following.

(a) $4a$, $6b$ (b) $3a$, $5ab$ (c) $8ab$, $12bc$

(d) $3x$, $5xy$, $15xy^2$ (e) $2ab$, $6abc$, $9ac$ (f) $4a^2b^3$, $8a^3b^2$, $10abc^2$

7. Express each of the following as a fraction with a single denominator.

(a) $\dfrac{x+1}{2}+\dfrac{x+3}{4}$ (b) $\dfrac{2x+1}{3}+\dfrac{2x-1}{6}$

(c) $\frac{a}{2} + \frac{2a-2}{5}$

(d) $x + \frac{2x-3}{3} + \frac{4-x}{4}$

(e) $5x - \frac{3x-4}{2} + \frac{x-7}{3}$

(f) $\frac{x-4}{3} - \frac{x-5}{4} - \frac{2x-3}{2}$

(g) $\frac{x}{3} - \frac{x-1}{6}$

(h) $\frac{2x-1}{4} - \frac{3x-4}{5}$

(i) $\frac{a}{3} + \frac{a-1}{6} - \frac{4a-5}{9}$

(j) $\frac{2a+1}{3} + \frac{3a-4}{4} - \frac{2a+1}{6}$

(k) $\frac{4xy}{3a} + \frac{xy}{a} - \frac{2xy}{5a}$

(l) $\frac{1}{2x} + \frac{3}{4x} - \frac{1}{6x}$

(m) $\frac{a+b}{2x} + \frac{3a-b}{3x} - \frac{b-a}{5x}$

(n) $\frac{2(x-y)}{z} + \frac{3(x+2y)}{4z} - \frac{5(x-4y)}{6z}$

8. Simplify each of the following.

(a) $\frac{x}{2} + \frac{x}{3} - \frac{x}{4}$

(b) $\frac{x-1}{2} - \frac{x-2}{3}$

(c) $\frac{x-4}{3} - \frac{x-5}{6} - \frac{3(x+1)}{5}$

(d) $\frac{4a}{2x+3} + \frac{6a}{4x+6}$

(e) $\frac{a+1}{x-2y} - \frac{2a-3}{2x-4y} + \frac{a}{3x-6y}$

(f) $\frac{x-3}{5x-4} - \frac{3(4-3x)}{4-5x}$

9. Simplify each of the following.

(a) $\frac{1}{2x+3y} + \frac{4x}{9y^2-4x^2} - \frac{2}{3y-2x}$

(b) $\frac{2}{x+4} + \frac{3}{4-x} - \frac{x}{x^2-16}$

(c) $\frac{2}{a+\frac{1}{2}}$

*(d) $\frac{\frac{1}{4}a}{2+\frac{1}{2}c}$

*(e) $\frac{1}{\frac{2}{a}+\frac{3}{b}}$

*(f) $\frac{\frac{2}{x}+\frac{5}{y}}{\frac{3}{x}}$

10. Simplify each of the following.

(a) $\frac{2}{3x+1} - \frac{1}{5x+3}$

(b) $\frac{3}{x+1} - \frac{x+4}{(x+1)(x+2)}$

(c) $\left(\frac{1}{x} - \frac{1}{y}\right) \div \left(\frac{1}{x^2} - \frac{1}{y^2}\right)$

(d) $\frac{1}{2(x-2)^2} + \frac{1}{x(x-2)}$

(e) $\frac{x}{x^2-4} - \frac{1}{x+2}$

(f) $\frac{1}{x-1} + \frac{2x}{1-x^2}$

(g) $\frac{2x}{x^2+x-6} + \frac{1}{x-2}$

(h) $\frac{1}{2(t-1)} + \frac{t+1}{t^2+t-2}$

(i) $\left(2x - \frac{8}{x}\right) \div \left(1 - \frac{2}{x}\right)$

(j) $\frac{2}{x-1} - \frac{1}{x-2} + \frac{3(x+2)}{(x-1)(x-2)}$

(k) $\frac{x+2}{x^2-1} - \frac{3}{2(x-1)}$

(l) $\frac{x}{x^2-4} - \frac{1}{x+2}$

(m) $\frac{x}{x+1} \div \frac{x^2-2x}{x^2-2x-3}$

(n) $\frac{x+2}{(x-3)(x-2)} - \frac{x}{(x+3)(x-3)}$

(o) $\frac{3}{x+2} - \frac{x}{x^2-4}$

(p) $\frac{4}{(x-2)(x-4)} - \frac{2}{(x-2)(x-3)}$

(q) $\frac{x^2}{(x+y)(x-3y)} - \frac{x-y}{x-3y}$

(r) $\frac{3x}{x-3} - \frac{x}{x^2-9}$

(s) $\frac{4}{(x-1)(x+3)} - \frac{1}{(x-4)(x-1)}$

(t) $\frac{x-4}{(x+1)(x-5)} - \frac{x+5}{(x+1)(x+3)}$

(u) $\frac{3x-2}{x^2-3x+2} - \frac{3x-1}{x^2-2x}$

11. Solve the following equations.

(a) $\frac{5}{x} = \frac{6}{7}$

(b) $\frac{7}{2x} = 3$

(c) $\frac{3}{x-2} = \frac{1}{2}$

(d) $\frac{5}{x-4} - 3 = 0$

(e) $\frac{9}{5-2x} + 7 = 0$

(f) $\frac{2x}{5x-4} + \frac{1}{3} = 0$

(g) $\frac{2}{5x} = \frac{4}{x-1}$

(h) $\frac{7}{2x-1} = \frac{3}{x-4}$

(i) $\frac{x+2}{3} = \frac{2x-1}{14}$

(j) $\frac{2a-5}{7} = \frac{3a+4}{9}$

(k) $\frac{3}{a+1} + \frac{1}{2a+1} = 0$

(l) $\frac{5}{2x-5} - \frac{4}{7x+1} = 0$

(m) $\frac{3}{1+2x} = \frac{5}{3+4x}$

(n) $\frac{4}{x} + 1\frac{1}{2} = \frac{5}{2x}$

(o) $\frac{10}{3x} - 2 = \frac{2}{3}$

(p) $\frac{3(x-1)}{2} + \frac{2x}{3} = 0$

(q) $\frac{2}{7x-9} - \frac{5}{6x-7} = 0$

(r) $\frac{3x}{10} + \frac{x-1}{2} = 0$

(s) $\frac{3x}{8} - \frac{x}{4} = \frac{1}{2}$

(t) $\frac{x}{3} - \frac{3(x+5)}{4} = \frac{3}{7}$

(u) $\frac{2x-3}{7} + \frac{3}{4} = \frac{5x-6}{2}$

12. Solve the following equations.

(a) $x = 8 - \frac{7}{x}$

(b) $x = \frac{3}{x+2}$

(c) $\frac{84}{x-4} = 1 + \frac{75}{x}$

(d) $x - 2 = \frac{9}{x-2}$

(e) $3 - x = \frac{8}{x+3}$

(f) $\frac{2x}{2x-3} + 1 = \frac{1}{2-3x}$

(g) $\frac{1}{x+2} + \frac{3}{x+4} = \frac{4}{x+3}$

(h) $\frac{3}{x+1} = \frac{8}{x+2} - \frac{5}{x+3}$

(i) $\frac{1}{x+3} + \frac{4}{5} = \frac{x}{4-x}$

(j) $\frac{5}{2x-7} - \frac{6}{x-7} = 0$

(k) $\frac{6x}{2x-1} = 2x$

(l) $\frac{x}{x-1} + \frac{x}{x+1} = 3 + \frac{1}{1-x^2}$

13. Solve the following equations.

(a) $\frac{2x}{3} - 5 = \frac{5x}{2} - 3$

(b) $\frac{x-1}{3} - \frac{5}{12} = x + \frac{1}{4}$

(c) $\frac{x+1}{3} - \frac{7}{18} = \frac{x}{4}$

(d) $\frac{1}{3}\left(\frac{1}{5x} - 3\right) = \frac{1}{2}\left(2 - \frac{1}{x}\right)$

(e) $\frac{1}{2}(5x+3) + 2 = \frac{1}{3}(1-2x)$

(f) $\frac{3}{5}(2x-5) = 1 - \frac{2}{5}(x+1)$

(g) $\frac{1}{3}(x-3)+3=x+2(x-1)$

(h) $x-\frac{1}{2}=1-\frac{x-1}{3}$

(i) $\frac{x-4}{3}-\frac{2x-1}{2}=4$

(j) $\frac{3}{x-4}-\frac{x-5}{(x-4)(2x+3)}=\frac{4}{2x+3}$

(k) $\frac{3t}{(t-1)(3t-2)}-\frac{5}{t-1}=\frac{3}{3t-2}$

(l) $\frac{x-2}{2}=\frac{4x+12}{3x}$

(m) $\frac{x+1}{x-3}=\frac{x+3}{x-7}$

(n) $\frac{4}{x-12}-\frac{4}{x}=\frac{1}{60}$

(o) $\frac{x}{x+3}=\frac{2x-4}{2x+9}$

(p) $\frac{3}{x+1}=\frac{2}{x^2-1}$

(q) $\frac{1}{x}=\frac{1}{20}-\frac{1}{x+30}$

(r) $\frac{x+1}{x-2}=\frac{3x-1}{x-1}$

(s) $\frac{1}{x-1}=\frac{1}{2x+1}+\frac{1}{2x-3}$

(t) $\frac{1}{x+1}-\frac{2}{x-2}=\frac{1}{3-x}$

14. Make a the subject of each of the following equations.

(a) $a+x=b$ **(b)** $a-k=h$ **(c)** $a-b=c+d$

(d) $a+c=d+e$ **(e)** $y+a=x$ **(f)** $z-a=2k$

(g) $p=a-q$ **(h)** $5k=p-a$ **(i)** $7k=p+a$

(j) $a-b-c=k^2$ **(k)** $b-a+k=h^3$ **(l)** $m+n+a=k$

(m) $m-n-a=h$ **(n)** $7k+h-a=2a$ **(o)** $5pq-a=p^2-q$

(p) $3xy+a=x^2y$ **(q)** $5a=15$ **(r)** $ax=3y$

(s) $2xy=3ak$ **(t)** $ak=p-q+k$ **(u)** $ax^2=5y-4$

15. Make a the subject of each of the following equations.

(a) $ah=b-c+k^3$ **(b)** $5ay^2=x^3-y$ **(c)** $ax=\frac{A}{b}$

(d) $3ak=\frac{15x}{7y}$ **(e)** $12mk=\frac{3ak^2}{5b+c}$ **(f)** $\frac{5a}{2y}=\frac{3kb}{4x}$

(g) $\frac{ae}{t}=\frac{t}{5n+4}$ **(h)** $\frac{x}{a+y}=\frac{y^2}{a}$ **(i)** $\frac{k(m+a)}{m}=\frac{4}{x}$

(j) $5(a-b)=7$ **(k)** $v=m(a+c)$ **(l)** $y=\frac{7ab+k}{7-4a}$

(m) $z=\frac{5-2a}{3-a}$ **(n)** $x=\frac{7+3a}{a-4}$ **(o)** $x=\frac{3ak+4xh+4}{5ab-4xy+2}$

16. Make the letters in the brackets the subject of the following formulae.

(a) $(x+p)a=q(2x-q)$ (x) **(b)** $\frac{an-5x}{3a-4x}=\frac{1}{3}$ (a) **(c)** $a=\frac{2b+c}{b}$ (b)

(d) $\frac{y-2x}{3y}=2x-7$ (x) **(e)** $T=\frac{4pr}{p+4s}$ (p) **(f)** $\frac{1}{v}+\frac{2}{u}=\frac{3}{f}$ (u)

(g) $x=\frac{12+5y}{4-y}$ (y) **(h)** $2a=\frac{5}{b}-\frac{4}{c}$ (b) **(i)** $\frac{ax+by}{3x-4y}=\frac{a}{b}$ (y)

(j) $y=\frac{ax+b}{cx+d}$ (x) **(k)** $\frac{1}{v}=\frac{u}{f}-1$ (f) **(l)** $yx-1=5(2x+3)$ (x)

(m) $\frac{F+40}{9}=\frac{c+40}{5}$ (c) **(n)** $P=\frac{ER}{k+R}$ (k) **(o)** $k=\frac{2x-1}{x+4}$ (x)

(p) $\frac{1}{x} + \frac{3}{2y} = \frac{4}{5z}$ (y) (q) $k - 3mx = \frac{3my}{4}$ (m) (r) $kx + 4 = \frac{2x - 3y}{2y - 5}$ (y)

(s) $\frac{3}{5} = \frac{y - 4a}{y + 7b}$ (y) (t) $\frac{a}{k} + h = \frac{b}{k}$ (k) (u) $\frac{1}{a} + \frac{2}{b} = \frac{3}{c} + \frac{4}{d}$ (b)

17. Make x the subject of each of the following equations.

(a) $y = \frac{2 - x^2}{2x^2 + 3}$ (b) $y = \sqrt{2x - 3y}$ (c) $\frac{1}{a} + \frac{1}{\sqrt{x}} = y$

(d) $y = \sqrt{\frac{(x^2 - 5)k}{2p}}$ (e) $k = \frac{2hax^2}{b - x^2}$ (f) $k = \sqrt{\frac{b(x - b)}{h}}$

(g) $\sqrt{x^2 - a^2} = x + a$ (h) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (i) $m = \frac{y - x}{yt - xz}$

(j) $v = \sqrt{gx + 3gh}$

18. Make the letter in the brackets the subject of each equation.

(a) $ax^2 + by + c = 0$ (y) (b) $k(lm + mn) = a$ (n)

(c) $t = 2\pi\sqrt{\frac{d}{g}}$ (g) (d) $\frac{1}{a} + \frac{2}{b} + \frac{3}{c} = \frac{4}{d}$ (b)

(e) $A = \pi r\sqrt{h^2 - r^2}$ (h) (f) $x = y + \frac{yk^2}{gm}$ (y)

(g) $v = \pi r^2 h + \frac{1}{3}\pi r^2 h$ (r) (h) $x = \sqrt{\frac{k^2 - t^2}{2k^2 + 3t^2}}$ (t)

19. If $u + v = m$ and $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, express m in terms of u and f.

20. If $y = p + \frac{q}{x}$ and $z = p + \frac{q}{y}$, express x in terms of p, q and z.

21. Two numbers differ by 5 and $\frac{3}{5}$ of the greater number is equal to $\frac{3}{4}$ of the smaller number. Find the numbers.

22. Two numbers differ by 9 and $\frac{4}{7}$ of the larger number is greater than $\frac{3}{4}$ of the smaller number by 3. Find the two numbers.

23. Find two consecutive numbers such that $\frac{4}{7}$ of the larger exceeds $\frac{1}{2}$ of the smaller by 5.

24. What number must be subtracted from both the numerator and the denominator of the fraction $\frac{29}{37}$ to make it equal to $\frac{5}{7}$? (b)

(u) 25. After spending $\frac{5}{8}$ of her money on marketing, a lady had \$10.50 left. How much had she at first?

(y) 26. Find three consecutive even numbers such that the sum of $\frac{3}{5}$ of the first, $\frac{1}{2}$ of the second and

(x) $\frac{3}{8}$ of the third is 32.

(x) 27. Find three consecutive odd numbers such that $\frac{1}{7}$ of the first plus $\frac{1}{3}$ of the second plus $\frac{1}{5}$ of the third is 63.

28. A man left half of a sum of money to his wife, one third of it to his son and the remainder to be divided equally between his twin daughters. If each of his daughters received $8500, how much was the original sum of money?

29. In 15 years' time, Kumar will be four times as old as he is now. How old is Kumar now?

30. When 8 is subtracted from $3\frac{3}{4}$ of a number, the result is 3 times the original number. Find the number.

31. John is $\frac{1}{4}$ as old as his father. In 8 years' time, the sum of their ages will be 61. How old is John's father?

32. Sarah spends $\frac{1}{7}$ of her time on Mathematics and $\frac{3}{7}$ of her time on languages in school daily. If she spends $\frac{3}{4}$ h on Mathematics, how many hours does she spend in school per day?

33. Devi saves $\frac{2}{5}$ of her pocket money each week, spends $\frac{3}{4}$ of the remainder on ice-cream and 70 cents on sweets. She finds that she still has 50 cents left. How much money does she receive each week?

34. A small hose running alone can fill a tank in 20 minutes, and a larger hose running alone can fill the same tank in 12 minutes.
(a) What fraction of the tank is filled by each hose running alone in one minute?
(b) What fraction is filled by both hoses running together for one minute?
(c) How many minutes will it take both hoses working together to fill up the tank?

35. When 4 is added to a number and the result divided by 5, the final result is 44 less than the original number. Find the number.

36. In a class, there are four more boys than girls. $\frac{1}{3}$ of the girls and $\frac{7}{11}$ of the boys take the train to school. If half of the class take the train, how many boys and girls are there in the class?

37. A motorist travelled 115 km on an expressway at an average speed of 92 km/h and another 45 minutes on other roads. If the average speed for his whole journey was 80 km/h, how far did he travel on the other roads?

38. At a tea-dance, there are 4 more girls than boys. For the first dance, only couples are allowed to take the floor, i.e. girls must dance with boys and girls are not allowed to dance together. If $\frac{5}{7}$ of the boys and $\frac{2}{3}$ of the girls take the floor for the first dance, how many boys are there altogether?

39. Divide 64 into two parts so that the difference between $\frac{2}{3}$ of one part and $\frac{1}{4}$ of the other is equal to 28. What is the value of the larger part?

40. What number must be added to both the numerator and the denominator of the fraction $\frac{7}{11}$ to give a result of $\frac{3}{4}$?

41. A pool can be filled with water by a large pipe within 6 hours. A smaller pipe will take 9 hours to fill the pool. How long will it take to fill the pool if the two pipes operate together?

42. A boy walks a certain distance at 9 km/h. He finds that if he increases his speed by $\frac{1}{3}$, he would have saved $1\frac{1}{3}$ hours. Find the distance he walks.

43. A cyclist travels a distance of 120 km from Town A to Town B at an average speed of x km/h. If he increases his average speed by 1 km/h, he would have saved 30 minutes. Find x.

Problem Posing: Problem Leading to Algebraic Fractions

Pose an interesting mathematical problem that will lead to an equation **involving algebraic fractions**. You will get more marks for creativity but the problem must be relevant and accurate. Solve the equation and ensure that the answers are relevant to the problem you posed.

My own mathematical problem and solution:

Scoring Rubric:

Competency Level	Mathematical Concept	Mathematical Accuracy	Creativity
4	Showed complete understanding of equations involving algebraic fractions	Obtained the correct answer by solving the equation accurately	Posed an original and very creative problem which was accurate; answers were relevant to the context
3	Showed nearly complete understanding of equations involving algebraic fractions	Made some minor errors in solving the equation	Posed an appropriate problem though not very creative; answers were relevant to the context
2	Showed some understanding of equations involving algebraic fractions	Made some errors in solving the equation	Posed an oversimplified problem but answers were still relevant to context
1	Showed limited understanding of equations involving algebraic fractions	Made some major errors in solving the equation	Posed an irrelevant problem where answers were also not relevant to the context
0	Showed no understanding of equations involving algebraic fractions	Did not solve the equation	No problem composed; slipshod and tardy work
Score			

Final Score:

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Final Score	10–12	8–9	6–7	4–5	0–3
Grade	A	B	C	D	F

Teacher's Comments (if any):

Chapter 5 Simultaneous Linear Equations

Summary

1. A pair of simultaneous linear equations in two variables can be solved by
   (a) elimination method,
   (b) substitution method,
   (c) graphical method.

2. Problems involving simultaneous equations may be solved by
   (a) assigning variables to unknowns,
   (b) forming the equations,
   (c) solving the equations,
   (d) giving the solution to the problem.

Practice Questions

Solve the following simultaneous equations.

1. $x + y = 7$
   $x - y = 3$

2. $x - 3y = 7$
   $x - y = 3$

3. $3x + y = 13$
   $5x - y = 35$

4. $3x + 2y + 7 = 0$
   $5x - 2y + 1 = 0$

5. $3x + w = 17$
   $3x - w = 19$

6. $3x + 2y = 8$
   $2y - 5x = 8$

7. $3x + 3 = 6y$
   $x - y = 1$

8. $3x = y + 1$
   $y - x = 3$

19. $5x - 3y = 23$
    $x - 7y = 11$

10. $15x - 7y = 14\frac{1}{4}$
    $5x - y = 3\frac{3}{4}$

11. $5x - 8y = 23\frac{1}{2}$
    $4x + y = 22\frac{1}{2}$

12. $4x - 6y = 12$
    $2x + 4y = -4.5$

13. $3x - 4y = 30$
    $2x - 7y = 33$

14. $3x - 5y = 19$
    $5x + 2y = 11$

15. $3x + 2y = 13$
    $5x - 4y = 18$

16. $4x - 2y = 5$
    $2x + 3y = -5$

17. $3x + 10y = 13$
    $24x - 36y = 17$

18. $3x - y = 7$
    $2x + 5y = -1$

19. $\frac{1}{2}x - \frac{1}{3}y - 1 = 0$
    $x + 6y + 8 = 0$

20. $3x - 2y = 8$
    $\frac{1}{8}x + \frac{1}{2}y = 1\frac{1}{4}$

21. $5x + y = 9$
    $2x - 3y = 7$

22. $4x - 3y = 8$

$6x + y = 1$

23. $x = 9 - 0.5y$

$y = 11 + \frac{1}{3}x$

24. $3x + 1.4y = 0.1$

$x - 3.6y = 10.2$

25. $2x - 1.6y = -0.8$

$0.5x + 1.2y = 7$

26. $2x + 0.5y = 6$

$3x - 0.25y = 5$

27. $0.5x - 0.2y = 2$

$2.5x + 0.6y = 2$

28. $x + \frac{1}{2}y = 9$

$3x - 2y = 13$

29. $x + 4y = 9$

$3x - 4y + 31 = 0$

30. $2y - 7x + 69 = 0$

$4x - 3y - 45 = 0$

31. $\frac{1}{2}x - \frac{1}{3}y = \frac{1}{4}$

$3x - y = 3$

32. $14x + 6y = 9$

$6x - 15y = -2$

33. $8x + 3y = 8\frac{1}{5}$

$15x - 10y = -10\frac{2}{3}$

34. $5x + 6y = -6\frac{1}{2}$

$7x - 9y = 2\frac{1}{2}$

35. $6x - 8y = 2\frac{2}{7}$

$7x - 5y = 4\frac{3}{14}$

36. $12x - 15y = 25\frac{1}{2}$

$17x - 7y = 29$

37. $7y - 4x = 1$

$15x - 9y = -9\frac{1}{2}$

38. $9x - 8y = 17\frac{2}{5}$

$3y - 5x = -8\frac{4}{5}$

39. $0.3x + 0.4y = 7$

$1.1x - 0.3y = 8$

40. $10x - 3y = 1$

$\frac{1}{4}x + \frac{1}{3}y = 1\frac{1}{4}$

41. $\frac{1}{5}x + \frac{3}{4}y = -1\frac{1}{2}$

$\frac{5}{6}x - \frac{1}{8}y = 13\frac{1}{4}$

42. $\frac{3}{2}x - \frac{2}{3}y = 1\frac{1}{6}$

$2x - 3y = 10$

43. $2x - 3y = 13$

$3x - 12y = 42$

44. $4x - 1\frac{5}{6} = 5y$

$2x + 3y = 9\frac{1}{6}$

45. $\frac{1}{2}x + 3y = -3\frac{3}{4}$

$4x - 5y = 8\frac{2}{3}$

46. $3x - 5y = 2$

$x - 2y = \frac{4}{15}$

47. $5x - 3y = 1.4$

$2x + 5y = 14.2$

48. $\frac{1}{3}x - \frac{2}{3}y + 5 = 0$

$\frac{1}{2}x + \frac{1}{3}y - \frac{1}{2} = 0$

49. $\frac{1}{3}(x + 1) + y = 8$

$x + 4 = \frac{1}{3}(y + 1)$

50. $3x - 4y - 7 = y + 10x - 10 = 4x - 7y$

51. $8x + 24 = 15x + 15y = 80 - 10y$

52. $4x + 4 = 5x = 60y - 100$

53. $x + y + 3 = 3y - 2 = 2x + y$

54. $5x + 3y = 29 = 2x + 7y$

55. $10x - 15y = 12x - 8y = 150$

56. $2x - 2 + 12y = 9 = 4x - 2y$

57. $5x - y + 19 = x - 2y = \frac{3}{4}x - \frac{2}{3}y + 3$

58. $5x - 8y = 3y - x + 8 = 2x - y + 1$

59. $4x + 2y = x - 3y + 1 = 2x + y + 3$

60. $\frac{2}{3}x - \frac{3}{5}y - 4 = \frac{1}{20}x - y + \frac{17}{30} = 2x - y - 18\frac{14}{15}$

61. $\frac{2}{7}x + \frac{3}{4}y - 4 = \frac{3}{5}x - \frac{2}{7}y - 44 = \frac{7}{15}x + y - 3\frac{1}{3}$

62. If $3x - 4y = 2$ and $2x + 5y = 9$, find $x : y$.

63. If $x = 3$ or -4 are the solutions of the equation $x^2 + ax + b = 0$, find the values of a and b.

64. If $x = -1$ and $y = 2$ are the solutions of the equations $ax - by = 1$ and $ay + bx = -7$, find the values of a and b.

65. Divide the number 32 into two parts so that one part is 3 times larger than the other.

66. One of the acute angles of a right-angled triangle is 15° larger than the other. Find the size of the larger acute angle.

67. The denominator of a fraction exceeds the numerator by 4. When 3 is added to both the numerator and the denominator, the fraction becomes $\frac{4}{5}$. Find the fraction.

68. The sum of two fractions is three times their difference. Six times the smaller fraction exceeds the larger fraction by $1\frac{1}{2}$. Find the fractions.

69. The sum of two numbers is 55. The quotient obtained by dividing the larger number by the smaller number is 2, with a remainder of 7. Find the two numbers.

70. Two ships leave the port at the same time and travel in opposite directions. The speed of the faster ship is 8 km/h more than the slower ship. At the end of $2\frac{1}{2}$ days, the two ships are 4320 km apart. Find the speed of the two ships.

71. A motorist covered a journey of 690 km in 8 hours. He covered part of the journey at 90 km/h and the rest at 80 km/h. What was the distance that he covered at 80 km/h?

72. Find two numbers such that if 7 is added to the greater, the answer is four times the smaller number while 28 added to the smaller number is equal to twice the greater number.

73. Find a fraction which reduces to $\frac{3}{4}$ when the numerator and denominator are each decreased by 1, and which reduces to $\frac{4}{5}$ when the numerator and denominator are each increased by 1.

74. 2 compact discs and one cassette tape cost \$52 while one compact disc and 5 cassette tapes cost \$53. Find the cost of a compact disc and a cassette tape.

75. 5 kg of beef and one chicken cost \$65 while 8 kg of beef and 6 chickens cost \$126. Find the cost of 1 kg of beef and a chicken respectively.

76. In a farm, there are some goats and some chickens. Paul counted 45 heads while Julie counted 150 legs. How many goats and how many chickens are there?

77. A farmer finds that he can buy 5 sheep and 5 cows for \$129 or 10 sheep and 17 cows for \$177. How much will it cost him to buy 3 sheep and 2 cows?

78. A housewife finds that 5 cans of condensed milk and 3 jars of instant coffee cost \$27 while 12 cans of condensed milk and 5 jars of instant coffee cost \$49.40. Find the total cost for 7 cans of condensed milk and 2 jars of instant coffee.

79. The bill for 6 cups of coffee and 7 cups of tea is \$5.90 while the bill for 18 cups of coffee and 5 cups of tea is \$9.70. Find the total bill for 7 cups of coffee and 6 cups of tea.

80. Aravind bought 25 stamps consisting of 20¢ and 50¢ stamps. If the total cost of these 25 stamps is \$8.60, find the number of each kind that he would have bought.

81. A man bought 8 kiwi fruits and 7 pears for \$4.10 while another man bought 4 kiwi fruits and 9 pears for \$3.70. What is the cost of each kiwi fruit and each pear?

82. In a factory, the technicians are paid \$24 per day and the packers \$20. If there are 540 workers and the total wage bill per day is \$12 000, find the number of technicians and the number of packers employed.

83. The ratio of two sums of money is 4 : 3. If the larger sum of money is increased by \$40, the ratio becomes 2 : 1. Find the sums of money.

84. The ratio of the ages of Elton and David is 3 : 5. In 6 years' time, the ratio of their ages will be 3 : 4. How old is David?

85. The sum of the ages of Mr Tan and his son David is 61 years. The difference in their ages is 29. How old is David? How old will Mr Tan be when David is 21 years old?

86. An aunt is 4 times as old as her nephew. In 8 years' time, the aunt will only be $2\frac{1}{2}$ times as old as the nephew. How old is the aunt now?

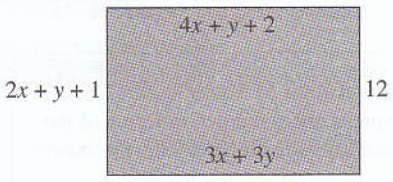
87. Ten years ago, a father was 8 times the age of his son. If the total of their ages is now 56, what are their present ages?

88. The sum of the ages of a mother and her daughter is 60 years. 12 years ago, the mother was eight times as old as her daughter. How old is the daughter now?

89. Robert is three times as old as Catherine. In 8 years' time, the ratio of their ages will be 2 : 1. How old is Catherine now?

Find the area and perimeter of each of the following rectangles. *The dimensions are in metres.*

90.



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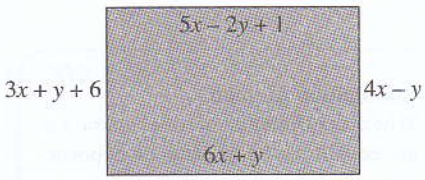

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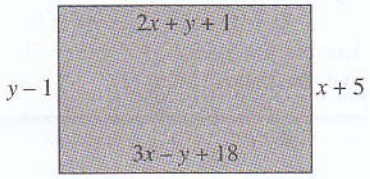

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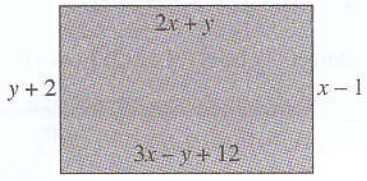

Find the perimeter of each of the following equilateral triangles. *The dimensions are in metres.*

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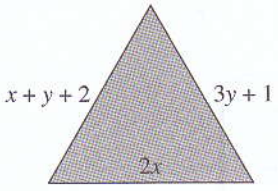

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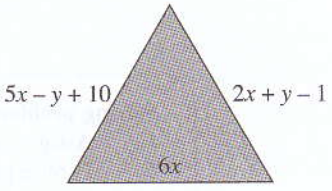

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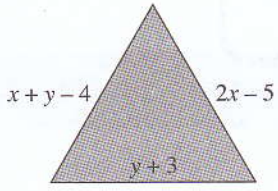

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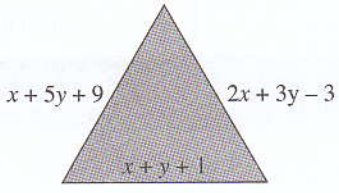
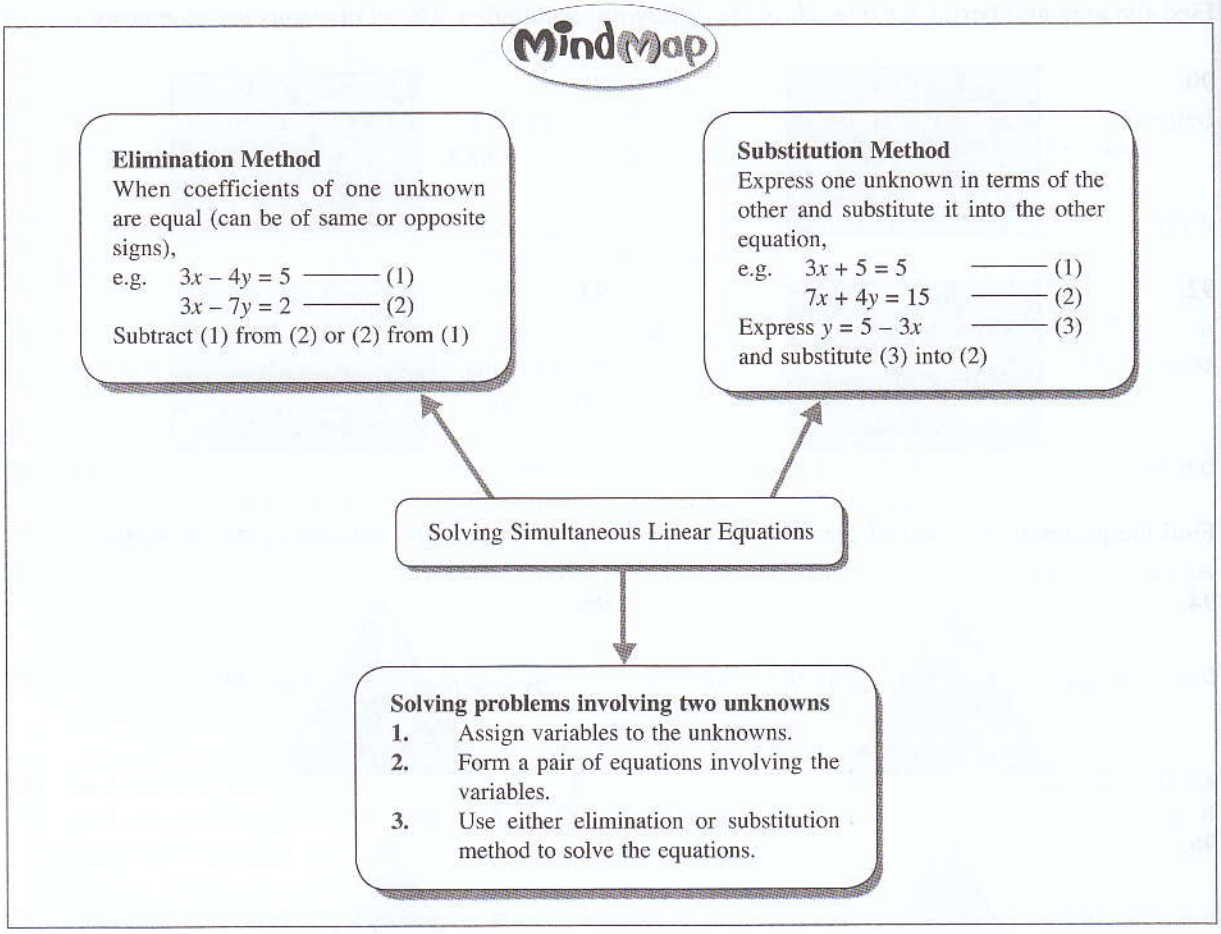


98. A two-digit number is 4 times the sum of its digits. If the digits are reversed, the number will be increased by 27. Find the number. [**Hint**: Let the number be $10x + y$.]

99. A man and his friend are 64 m apart. They will meet in 8 seconds if they walk towards each other. If they walk in the same direction, the man will catch up with his friend in 32 seconds. Find the speed of the man and his friend.

100. The 'tens' digit of a two-digit number is half of the 'units' digit. When the digits are reversed, the number is increased by 36. Find the number.

MindMap


Elimination Method
When coefficients of one unknown are equal (can be of same or opposite signs),
e.g. 3x – 4y = 5 ——— (1)
3x – 7y = 2 ——— (2)
Subtract (1) from (2) or (2) from (1)
Substitution Method
Express one unknown in terms of the other and substitute it into the other equation,
e.g. 3x + 5 = 5 ——— (1)
7x + 4y = 15 ——— (2)
Express y = 5 – 3x ——— (3)
and substitute (3) into (2)
Solving Simultaneous Linear Equations
Solving problems involving two unknowns
1. Assign variables to the unknowns.
2. Form a pair of equations involving the variables.
3. Use either elimination or substitution method to solve the equations.

Alternative ASSESSMENT

Problem Posing: Simultaneous Linear Equations

Two simultaneous equations are given as follows:

$$8x + 3y = 14$$
$$2x + y = 4$$

Compose a problem that will lead to the formation of the above two linear equations in two unknowns. Then solve the above simultaneous equations using both elimination and substitution methods. Make sure that the answers you get are appropriate for the problem that you have composed.

Problem:

Solution:

Elimination Method	Substitution Method

Scoring Rubric:

Competency Level	Mathematical Concept	Mathematical Accuracy	Originality
4	Showed complete understanding of simultaneous equations	Obtained the correct answer using both elimination and substitution methods accurately	Posed an original and very creative problem which was accurate; answers were relevant to the context
3	Showed nearly complete understanding of simultaneous equations	Made some minor errors in solving the simultaneous equations	Posed an appropriate problem though not very creative; answers were relevant to the context
2	Showed some understanding of simultaneous equations	Made some errors in solving the simultaneous equations	Posed an oversimplified problem but answers were still relevant to context
1	Showed limited understanding of simultaneous equations	Made some major errors in solving the simultaneous equations	Posed an irrelevant problem where answers were also not relevant to the context
0	Showed no understanding of simultaneous equations	Did not solve the simultaneous equations	No problem composed; slipshod and tardy work
Score			

Final Score:

[ ] / 12

Final Score	10–12	8–9	6–7	4–5	0–3
Grade	A	B	C	D	F

Teacher's Comments (if any):

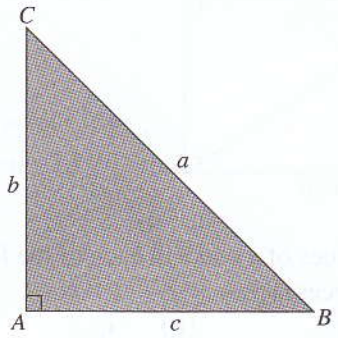
Chapter 6 Pythagoras' Theorem

Summary

1. Pythagoras' Theorem:
For a right-angled triangle ABC,

$$BC^2 = AC^2 + AB^2$$

i.e., $a^2 = b^2 + c^2$

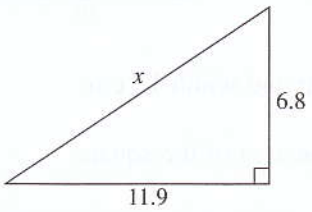


2. The converse of the Pythagoras' Theorem states that if $a^2 = b^2 + c^2$, then the triangle with sides a, b and c is a right-angled triangle, with the angle opposite the side a being a right angle.

Practice Questions

1. Calculate the value of x in each of the following, giving your answer correct to 3 significant figures.

(a)



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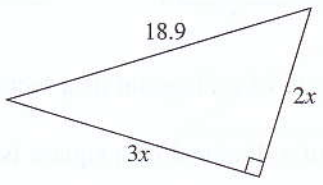

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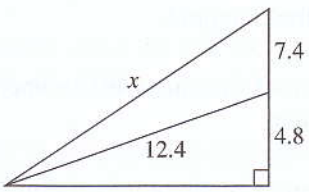

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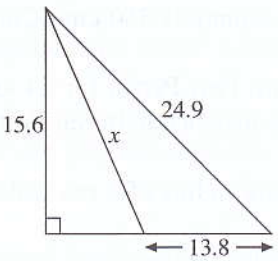

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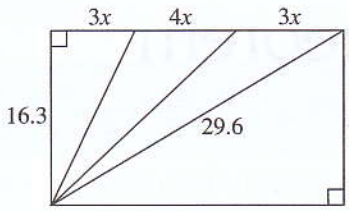

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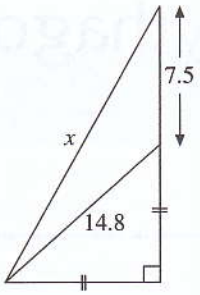

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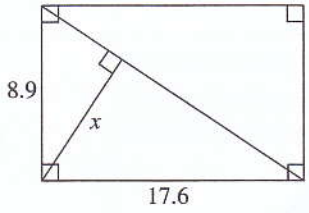

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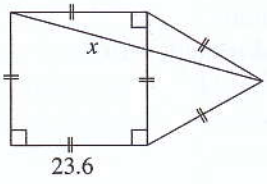
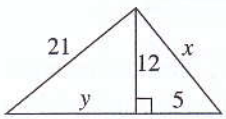


2. Calculate the values of x and y in each of the following, giving your answer correct to 3 significant figures where necessary.

(a)



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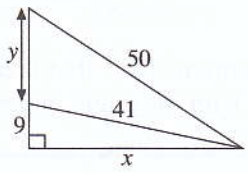

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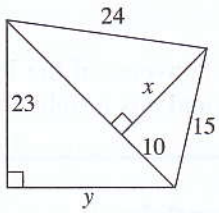

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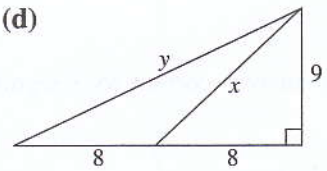

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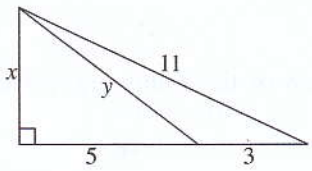
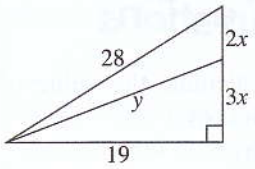


(f)



3. Find the length of a diagonal of a rectangle of length 14 cm and width 12 cm.

4. The length of a diagonal of a square is 12 cm. Calculate the area of the square.

5. A square has diagonals of length 22 cm. Calculate the perimeter of the square.

6. The area of the square is 350 cm^2. Calculate the length of the diagonal.

7. A ship sails from Port Perrin for 24 km due north and then 45 km due west to anchor at Cape Logan. Find the distance between Port Perrin and Cape Logan.

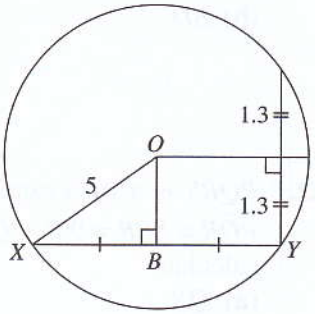
8. A cone has a base radius of 8 cm and a slant height of 12 cm. Calculate its vertical height.

9. An equilateral triangle has side 8 cm. Calculate its altitude and area.

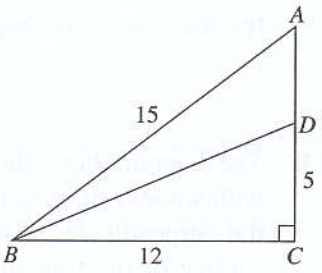
10. An aircraft flies 240 km south and then 140 km west. How far is the aircraft from its starting point?

11. A ladder leans against a vertical wall and reaches to a height of 3.2 m. If the foot of the ladder is 0.8 m from the wall, calculate the length of the ladder.

12. Calculate the length of XY in the given figure (measurements are in cm).



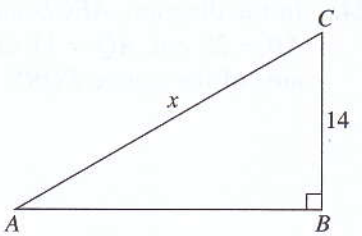
13. ABC is a right-angled triangle with $A\hat{C}D = 90°$, $BC = 12$ cm, $AB = 15$ cm and $CD = 5$ cm. Find the lengths of BD and AD.



14. Two vertical posts are 14 m apart. One is 3 m high and the other is 4.6 m high. Find the distance between the tops of the two posts.

15. A ship sails 29 km north from a port K to a port P. Then it sails 21 km towards the east to a port Q. Calculate the distance between port K and port Q.

16. In $\triangle ABC$, $AC = x$ cm, $BC = 14$ cm and the area of $\triangle ABC = 180$ cm^2. Calculate the value of x.



17. The diagonals of a rhombus are 28 cm and 54 cm. Calculate the length of its sides, giving your answer correct to 4 significant figures.

18. In $\triangle ABC$, $AB = 15$ cm, $AC = 18$ cm and AD is perpendicular to BC so that $AD = 12$ cm. Find the length of BC.

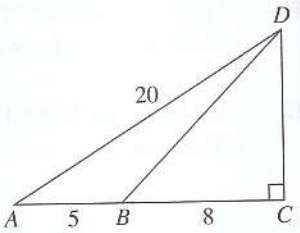
19. $\triangle ABC$ is an isosceles triangle where $AB = AC = 17$ cm and $BC = 16$ cm. Calculate the area of $\triangle ABC$.

20. A rhombus of side 32 cm has a diagonal of 50 cm. Find the length of the other diagonal.

21. In the diagram, $AB = 5$ cm, $BC = 8$ cm, $AD = 20$ cm and $A\hat{C}D = 90°$. Calculate the length of

(a) CD,

(b) BD.

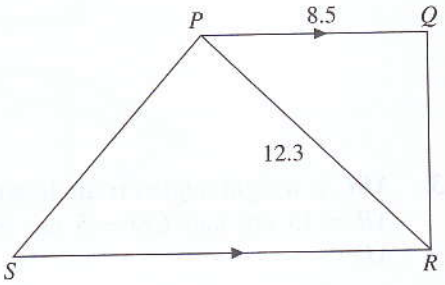


22. $PQRS$ is a trapezium where PQ // SR. Given that $P\hat{Q}R = S\hat{P}R = 90°$, $PQ = 8.5$ cm and $PR = 12.3$ cm, calculate

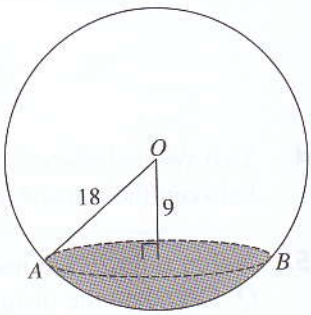
(a) QR,

(b) PS,

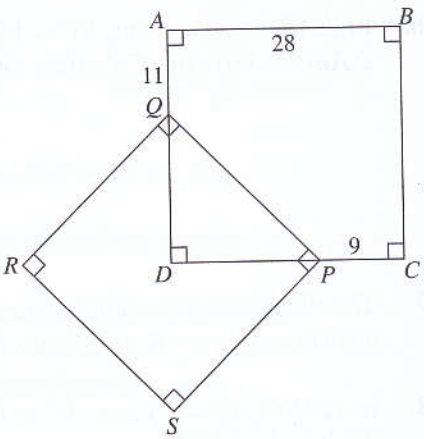
(c) the area of the trapezium $PQRS$.



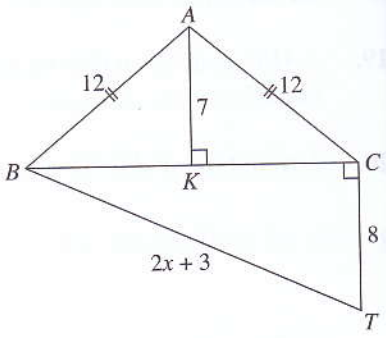
23. The diagram shows the cross-section of a clear spherical container with some fruit juice in it. The radius of the sphere is 18 cm and the perpendicular distance from the centre of the sphere to the surface of the fruit juice is 9 cm, calculate the diameter of the surface of the fruit juice in the sphere.



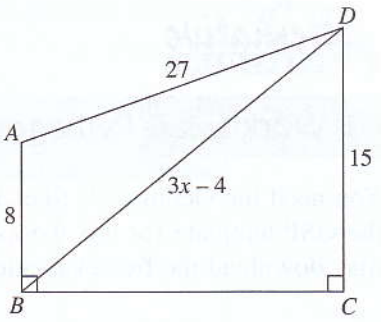
24. In the diagram, $ABCD$ and $PQRS$ are squares. Given that $AB = 28$ cm, $AQ = 11$ cm and $PC = 9$ cm, calculate the area of the square $PQRS$.



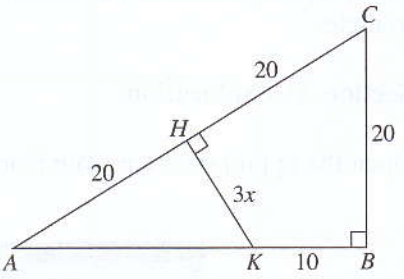
25. In the diagram, ABC is an isosceles triangle, $A\hat{K}B = B\hat{C}T = 90°$. Given that $AB = AC = 12$ cm, $AK = 7$ cm, $CT = 8$ cm and $BT = (2x + 3)$ cm. Calculate the value of x, giving your answer correct to 2 decimal places.



26. In the diagram, $A\hat{B}D = B\hat{C}D = 90°$. Given that $AB = 8$ cm, $CD = 15$ cm, $AD = 27$ cm and $BD = (3x - 4)$ cm, calculate the value of x, giving your answer correct to 2 decimal places.



27. In the diagram, $A\hat{B}C = C\hat{H}K = 90°$. Given that $CH = AH = BC = 20$ cm, $KB = 10$ cm and $HK = 3x$ cm. Calculate the value of x.



28. The lengths of the sides of a right-angled triangle are $(x + 1)$ cm, $4x$ cm and $(4x + 1)$ cm. Calculate the value of x.

29. The lengths of the sides of a right-angled triangle are x cm, $(x + 2)$ cm and $(x + 4)$ cm. Calculate the value of x and hence find the area of the triangle.

30. The lengths of the sides of a right-angled triangle are $2x$ cm, $(4x - 1)$ cm and $(4x + 1)$ cm. Calculate the value of x and hence find the perimeter of the triangle.

Alternative ASSESSMENT

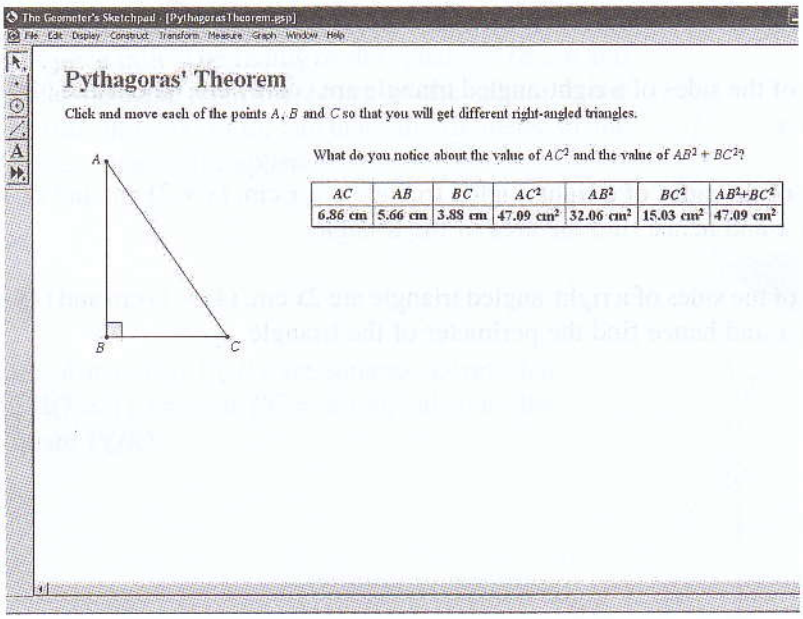
IT Worksheet: Pythagoras' Theorem

You need the Geometer's Sketchpad (GSP), a dynamic geometry software, to view and interact with the GSP template for this worksheet. If your school does not have a licensed copy of version 4, you may download the free evaluation version from www.keypress.com for trial first.

The purpose of this worksheet is to explore the relationship among the sides of a right-angled triangle.

Section A: Exploration

Open the appropriate template from the Workbook CD.



1. The template shows a **right-angled** triangle *ABC*. The longest side of a right-angled triangle is called the **hypotenuse**. Which side of $\triangle ABC$ in the template is the hypotenuse? **[1]**

2. Click and move each of the points A, B and C so that you will get five different right-angled triangles. Complete the table below. [3]

No.	AC	AB	BC	AC^2	AB^2	BC^2	$AB^2 + BC^2$
1							
2							
3							
4							
5							

3. What do you notice about the value of AC^2 and the value of $AB^2 + BC^2$ in the table above? [2]

Section B: Animation

- Right-click on the table in the template and select: ***Add Table Data...***
- Select the second option: ***Add 10 Entries As Values Change, Adding 1 Entry Every 1.0 Second(s)***. Click ***OK***.
- Select the vertex B of the triangle.
- Choose from the Toolbar: ***Display*** ▸ ***Animate Point or Animate Objects***.

Section C: Conclusion

4. The relationship that you have discovered in Q3 is called the **Pythagoras' Theorem**. State the Pythagoras' Theorem in the box below. Remember to specify clearly which is the hypotenuse. [1]

5. Do you think Pythagoras' Theorem is still true if the triangle is not a right-angled triangle? [1]

6. What is the geometrical significance of Pythagoras' Theorem? That is, try to represent the theorem using a right-angled triangle and some squares in the box below and write the theorem in terms of the areas of the squares. [2]

Section D: Enrichment

7. Although this theorem is named after the Greek mathematician Pythagoras (about 569–475 BC), it was actually discovered by other people first, e.g. the Chinese. Search the Internet to find out who discovered this theorem before Pythagoras. [1]

8. There are hundreds of proofs of Pythagoras' Theorem. Some of these proofs are easier to understand than others. Search the Internet to find out one proof that you can understand. Then write down the proof, with diagrams, in the space below. [4]

Final Score:

_____ / 15

Final Score	12–15	10–11	8–9	6–7	0–5
Grade	A	B	C	D	F

Teacher's Comments (if any):

Mathematical Investigation: Pythagorean Triples

Three natural numbers, a, b and c, form a set of Pythagorean triples if they are the lengths of the sides of a right-angled triangle, i.e. $a^2 = b^2 + c^2$ if a is the length of the longest side or hypotenuse of the right-angled triangle. For example, {3, 4, 5} is a set of Pythagorean triples because $3^2 + 4^2 = 5^2$. The objective of this investigation is to find some Pythagorean triples using three methods.

Section A: How to Generate Pythagorean Triples using an Odd Number

1. Start with an odd number, e.g. 3. Then the square of 3 can be expressed as the sum of two consecutive numbers, i.e. $3^2 = 9 = 4 + 5$. Therefore, the Pythagorean Triples are {3, 4, 5} since $3^2 + 4^2 = 25 = 5^2$. Generate another set of Pythagorean Triples starting with the next odd number 5 and verify that it works. **[2]**

2. Generate another set of Pythagorean Triples starting with the next odd number 7 and verify that it works. **[2]**

3. Prove that this method will always work for any odd number except 1. Explain why this method does not work for the odd number 1. **[4]**

Section B: How to Generate Pythagorean Triples using an Even Number

4. Start with an even number, e.g. 6. Then the square of half of 6 is $3^2 = 9$. So the other two numbers are 8 and 10 (one more than 9 and one less than 9). Therefore, the Pythagorean Triples are {6, 8, 10} since $6^2 + 8^2 = 100 = 10^2$. Generate another set of Pythagorean Triples starting with the next even number 8 and verify that it works. **[2]**

5. Generate another set of Pythagorean Triples starting with the next even number 10 and verify that it works. **[2]**

6. Prove that this method will always work for any even number except 2. Explain why this method does not work for the even number 2. **[4]**

Section C: How to Generate Pythagorean Triples from a Primitive Set

7. You may have noticed that each of the Pythagorean Triples {6, 8, 10} in Q4 is two times the Pythagorean Triples {3, 4, 5} in Q1. {3, 4, 5} are called **primitive** Pythagorean Triples because the highest common factor (HCF) of the three numbers is 1 and you cannot reduce them further. You can obtain {6, 8, 10} from {3, 4, 5} by multiplying each number by 2. Explain why {6, 8, 10} form a set of Pythagorean Triples if {3, 4, 5} is a set of Pythagorean Triples. **[2]**

8. Generate another two sets of Pythagorean triples from the primitive Pythagorean triples {3, 4, 5}. **[1]**

9. Explain why {5, 12, 13} is a set of primitive Pythagorean triples. Then generate two sets of Pythagorean triples from it. **[2]**

10. Is {16, 30, 34} a set of primitive Pythagorean triples? Why or why not? Then generate two sets of Pythagorean triples that are related to it. **[2]**

Section D: Conclusion

11. Write down one main lesson that you have learnt from this worksheet. **[1]**

"Numbers rule the universe." (Pythagoras of Samos, ~569-475BC)

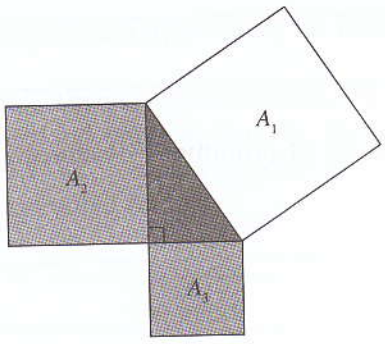
Final Score: ☐ / 24

Final Score	20–24	16–19	12–15	8–11	0–7
Grade	A	B	C	D	F

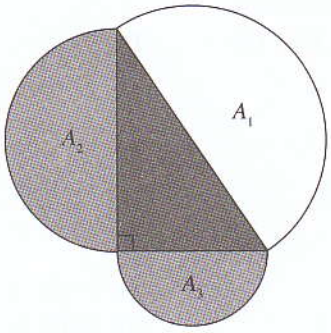
Teacher's Comments (if any):

Mathematical Investigation: Generalised Pythagoras' Theorem

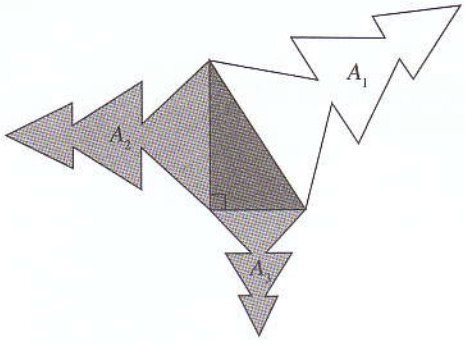
- Pythagoras' Theorem is usually stated in terms of the sides of a right-angled triangle: $a^2 + b^2 = c^2$.
- But it can also be stated in terms of areas (see diagram below): Area A_1 = Area A_2 + Area A_3.



1. What if the areas do not refer to the areas of squares but areas of semicircles (see diagram below)? Does the relationship still work? That is, is A_1 still equal to $A_2 + A_3$? Show your working clearly in the space provided. **[4]**



2. What if the areas refer to the areas of any **similar** shapes (see diagram below)? Does the relationship still work? That is, is A_1 still equal to $A_2 + A_3$? Show your working clearly in the space provided. **[4]**



Note: This relationship is called the **Generalised Pythagoras' Theorem**.

"Everything is arranged according to numbers and mathematical shapes." (Pythagoras, ~569-475BC)

Final Score:

[ ] / 8

Final Score	7–8	6	4–5	3	0–2
Grade	A	B	C	D	F

Teacher's Comments (if any):

Term II Revision Test

Time: $1\frac{1}{2}$ h

1. Factorise the following.
   (a) $24s^2 - 13s - 2$
   (b) $64a^2 - 25b^2 - (8a - 5b)$ [4]

2. Solve the following equations.
   (a) $5x + 15(x - 4) = 2(x - 3)$
   (b) $(y - 2)^2 - 25 = 0$ [4]

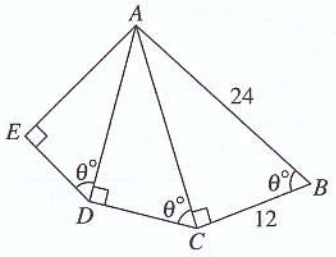
3. (a) Given that $(2p - q)(r + 5) = r(p - 1)$, express r in terms of p and q. Find the value of r given that $p = 6$ and $q = -3$. [3]
   (b) Solve the equations $2x - y = 1$ and $8x - 3y = 9$. [3]

4. Express the following as fractions with a single denominator.
   (a) $\frac{2y+1}{5} - \frac{3y-2}{10} + \frac{y}{2}$
   (b) $\frac{k}{u-k} - \frac{v}{u-v}$
   (c) $2 - \frac{1}{1-\frac{1}{b}}$ [6]

5. Simplify the following.
   (a) $\frac{\frac{3}{x} - \frac{x}{3}}{\frac{1}{x} - \frac{1}{3}}$
   (b) $\frac{\frac{6}{y} - 5 + y}{\frac{1}{2} + \frac{1}{2y} - \frac{3}{y^2}}$
   (c) $\frac{10u^2v + 2uv^2}{2u^2v}$ [6]

6. Solve the following equations.
   (a) $\frac{6t-3}{2t+7} = \frac{3t-2}{t+5}$
   (b) $6x - 4 = \frac{2}{x}$ [4]

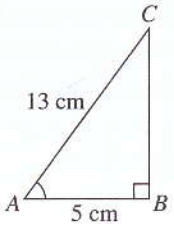
7. In the diagram, $A\hat{C}B = A\hat{D}C = A\hat{E}D = 90°$, $A\hat{B}C = A\hat{C}D = A\hat{D}E = \theta°$, $BC = 12$ cm and $AB = 24$ cm. Calculate, giving your answers correct to 2 decimal places, the length of
   (a) ED, (b) AE. [4]



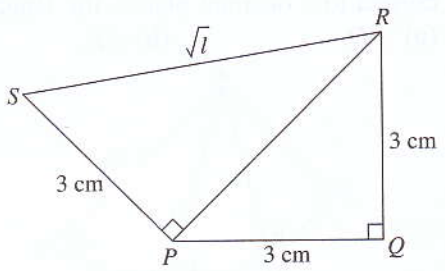
8. (a) Given that $2a = x^2 - bt$, express t in terms of a, b and x. Find the value of t when $x = 4$, $a = -6$, and $b = -7$. [3]
   (b) Simplify
   (i) $\frac{9k-15}{9k^2-25}$,
   (ii) $\frac{(3x-2)(x-2)-5x}{x-4}$. [4]

9. The resistance (R newtons) to the motion of a car is given by the equation
   $$R = a + bV^2,$$
   where V km/h is the speed of the car and a and b are constants. The resistance is 281 newtons when the speed is 27 km/h and 344 newtons when the speed is 36 km/h.
   (a) Form two equations in a and b.
   (b) Solve these equations to find the value of a and the value of b.
   (c) Calculate
   (i) the resistance when the speed is 63 km/h,
   (ii) the speed when the resistance is 425 newtons. [7]

10. (a) In the diagram, calculate the length of BC and find the area of $\triangle ABC$. [3]



(b) In the diagram, given that $PQ = QR = PS = 3$ cm and $SR = \sqrt{l}$ cm, find the value of l. **[3]**

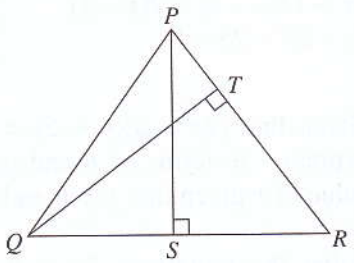


11. In the diagram, PS is perpendicular to QR and QT is perpendicular to PR. Given that $SR = 5$ cm, $PR = 13$ cm and $QR = 14$ cm, calculate

(a) PS, **(b)** PQ,

(c) the area of $\triangle PQS$.

Hence show that $QT = 12\frac{12}{13}$ cm. **[6]**



Mid-Year Specimen Paper

Answer **all** *the questions.* **Time: $2\frac{1}{2}$ h**

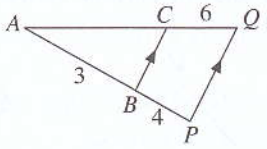
1. A man jogs 1200 m in 6 min. Express his average speed in kilometres per hour. **[2]**

2. Solve the equation
   (a) $5x(x - 3) = 0$,
   (b) $6x^2 + x - 1 = 0$. **[2]**

3. Factorise completely
   (a) $5x^2 - 20$,
   (b) $2ac - 2bc - bd + ad$. **[2]**

4. Given that $(a + b)^2 = 189$ and $6ab = 78$, calculate the value of $3(a^2 + b^2)$. **[3]**

5. In the diagram, BC is parallel to PQ. It is given that $AB = 3$ cm, $BP = 4$ cm and $CQ = 6$ cm. Find the length of AC. **[2]**



6. Solve the simultaneous equations $3x - y = 13$ and $\frac{x}{3} - \frac{y}{4} = 1$. **[3]**

7. Given that $3x - y = ax + b$, express x in terms of a, b and y. **[2]**

8. Map A is drawn to a scale of 2 cm to 5 km while map B is drawn to a scale of 3 cm to 4 km. A forest is represented by an area of 72 cm^2 on map A. Find its area represented on map B. **[2]**

9. **(a)** Expand and simplify
   $(2x - y)(x + 3y) - x(2x - 3y)$. **[2]**
   (b) Express the following as a single fraction in its simplest form.
   $$\frac{2}{3x + 4y} + \frac{3x}{16y^2 - 9x^2} - \frac{5}{3x - 4y}$$ **[3]**

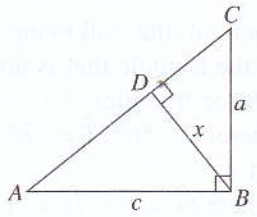
10. The resistance (R) of a wire of constant length is inversely proportional to the square of its diameter. If the resistance of 1 kilometre of copper wire 2 mm in diameter is 23 ohms, find the resistance of another 1 kilometre length of copper wire with diameter 2.3 mm. **[3]**

11. A ladder 8.5 m long leans against a vertical wall. If the foot of the ladder is 2.4 m from the bottom of the wall, find how high the ladder reaches up the wall. **[2]**

12. John and David set off from the same point. John walks 25 m south-east to a point P and David walks 34 m south-west to a point Q. Find the length of PQ. **[3]**

13. Simplify each of the following.
   (a) $\frac{x}{3} - \frac{x - 2}{6}$ **(b)** $\frac{5}{x} - \frac{7}{xy}$ **[3]**

14. In the figure, $A\hat{B}C = B\hat{D}C = 90°$, $AB = c$ cm, $BC = a$ cm and $BD = x$ cm. Name a pair of similar triangles.
   Find the length of CD in terms of x, a and c. **[3]**



15. The perimeter of a rectangle is given by $P = 2(B + L)$.
   (a) Find L when $B = 6$ cm and $P = 42$ cm.
   (b) Make B the subject of the formula. **[3]**

16. **(a)** Factorise $a^2 - b^2$.
   (b) Use the result in **(a)** to evaluate $88.74^2 - 11.26^2$. **[3]**

17. A woman bought 7 pineapples and 9 mangoes for \$19.10. Later in the day, she bought 3 pineapples and 6 mangoes for \$11.40. How much is each pineapple and each mango? **[4]**

18. Simplify $\left(1 - \frac{25}{4x^2}\right) \div \left(1 - \frac{5}{2x}\right)$. **[2]**

19. The length and breadth of a rectangle are $(2x + 1)$ cm and $(x - 1)$ cm respectively. If the area of the rectangle is 90 cm^2, form an equation in x and hence calculate the value of x and find the length of the diagonal of the rectangle. **[6]**

20. Given that $\frac{x-1}{2x+3} = y + 4$, express x in terms of y. **[4]**

21. A map is drawn using a scale of 4 cm to represent 5 km.

(a) Find the representative fraction. **[1]**

(b) A road has a length of 9.4 cm on the map. Find its actual distance in km. **[1]**

(c) A lake has an area of 64 km^2. Find its area represented on the map. **[2]**

22. (a) Factorise $2a^2 - ap - 2ac + pc$.

(b) Solve the equation $2x^2 + 5x - 12 = 0$. **[4]**

23. The sum of the reciprocals of $(3 - x)$ and $(1 - 2x)$ is 6 times the reciprocal of $(3 - 4x)$. Find the value of x. **[4]**

24. Sketch each of the following triangles and single out the triangle that is not congruent to the other three triangles.

$\triangle CAT$ where $\hat{A} = 56°$, $\hat{T} = 78°$ and $AT = 9$ cm

$\triangle DOG$ where $\hat{O} = 46°$, $\hat{D} = 78°$ and $DG = 9$ cm

$\triangle RUN$ where $\hat{R} = 78°$, $\hat{N} = 56°$ and $RU = 9$ cm

$\triangle PIE$ where $\hat{I} = 56°$, $\hat{E} = 46°$ and $IP = 9$ cm. **[4]**

25. Solve the following equations.

(a) $3x - 4 - 7(3 - 2x) = 0$ **[1]**

(b) $x(y + 2) - 3(y + 2) = 0$ **[2]**

(c) $(8x - 5)^2 = 98 - (x + 9)^2$ **[3]**

26. (a) In a canteen, there are a number of four-legged chairs and three-legged stools. David counted 40 seats and Daniel counted 145 legs. How many chairs and stools are there? **[4]**

(b) Explain why the following pair of simultaneous equations have no solution:

$x - 2y = 3$, $6y - 3x = 4$. **[1]**

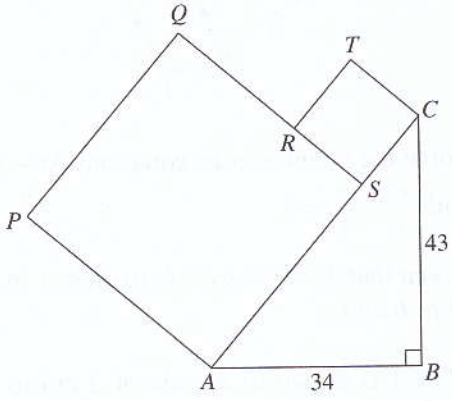
27. Mr Lim's car can travel x km for every litre of petrol used. He makes a journey of 480 km to Malaysia. Write down the number of litres of petrol used to travel 480 km.

Mr Tan's car can travel $(x - 2)$ km for every litre of petrol used. Write down the number of litres of petrol used to travel 480 km.

Given that Mr Tan's car uses 8 litres of petrol more than Mr Lim's car, form an equation in x.

Hence, find the number of litres of petrol used by Mr Lim's car. **[5]**

28. In the diagram, $APQS$ and $RSCT$ are squares. Given that $A\hat{B}C = 90°$, $AB = 34$ cm, $BC = 43$ cm and area of the square $RSCT$ is 250 cm^2, calculate the area of the square $APQS$. Give your answer correct to the nearest whole number. **[4]**



29. Two quantities p and q vary such that $p = a + bq$, where a and b are constants. Given that $p = 6$, when $q = \frac{1}{6}$ and $p = 10$ when $q = \frac{1}{3}$,

(a) write down two equations in a and b,

(b) solve these equations to find the value of a and the value of b,

(c) find

(i) the value of p when $q = 2$,

(ii) the value of q when $p = 0$. **[6]**

Chapter 7 Volume and Surface Area

Summary

1. For a pyramid and a prism with the same polygonal base and the same height,

$$\text{Volume of Pyramid} = \frac{1}{3} \times \text{Volume of Prism}$$
$$= \frac{1}{3} \times \text{Base Area} \times \text{Height}$$

2. For a cone and a cylinder with the same circular base of radius r and the same height h,

$$\text{Volume of Cone} = \frac{1}{3} \times \text{Volume of Cylinder}$$
$$= \frac{1}{3} \times \text{Base Area} \times \text{Height}$$
$$= \frac{1}{3}\pi r^2 h$$

3. Total Surface Area of Pyramid = Sum of Areas of All the Faces

4. For a cone with circular base of radius r and slant height l,

$$\text{Curved Surface Area of Cone} = \pi rl$$
$$\text{Total Surface Area of Cone} = \pi rl + \pi r^2 = \pi r(l + r)$$

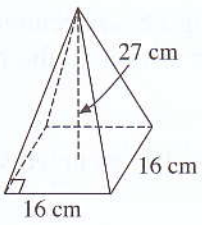
5. For a sphere with radius r,

$$\text{Volume of Sphere} = \frac{4}{3}\pi r^3$$
$$\text{Surface Area of Sphere} = 4\pi r^2$$

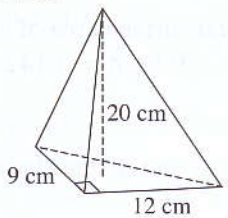
Practice Questions

Find the volumes of the following pyramids.

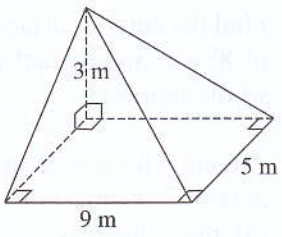
1.



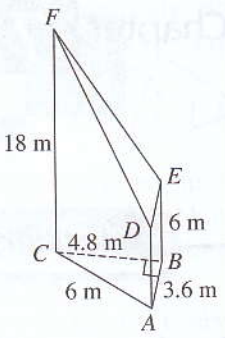
2.



3.



4. The diagram shows a solid made up of a pyramid and a prism whose base is a right-angled triangle.
If $CF = 18$ cm, $AD = BE = 6$ m, $AB = 3.6$ m, $BC = 4.8$ m and $CA = 6$ m, find the volume of the solid.



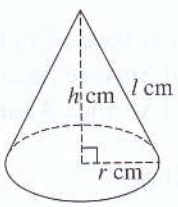
5. If the volume of a pyramid is 42 cm^3 and the base area is equal to 8 cm^2, find the height of the pyramid.

6. A pyramid has a rectangular base measuring 8 m by 3 m. Its volume is 86 m^3. Find its height.

7. A pyramid has a right-angled triangular base and a volume of 160 cm^3. The lengths of the two shorter sides of the triangular base are 12 cm and 5 cm. Find the height of the pyramid.

8. Find the vertical height of a pyramid with volume 84 cm^3 and a rectangular base 9 cm by 6 cm.

The diagram shows a right circular cone whose height is h cm, base radius r cm and slant height l cm.
Taking $\pi = 3.142$, find **(a)** the volume (correct to the nearest cm^3), and **(b)** the total surface area (correct to the nearest cm^2) of the cone when



9. $r = 6$, $h = 8$, $l = 10$,
10. $r = 12$, $h = 9$, $l = 15$,
11. $r = 12$, $h = 28.8$, $l = 31.2$,
12. diameter = 48, $h = 10$, $l = 26$,
13. diameter = 28.8, $h = 27$, $l = 30.6$.

The volume of a right circular cone, whose base radius is r cm and height h cm, is V cm^3.
Taking $\pi = 3.142$, find the value of h correct to three significant figures when

14. $r = 8$, $V = 320$,
15. $r = 5$, $V = 245$,
16. $r = 10.6$, $V = 342.8$,
17. diameter = 22, $V = 368$.

Taking $\pi = 3.142$, find the value of r correct to two significant figures when

18. $h = 6$, $V = 254$,
19. $h = 11$, $V = 695$,
20. $h = 17$, $V = 498$.

21. Find the curved surface area and the total surface area of a solid cone having a base circumference of 88 cm and a slant height of 15 cm. (Take $\pi = 3.142$. Round off your answer to the nearest whole number.)

22. A cone with a base radius of $(x - 5)$ cm and a slant height of $(x + 5)$ cm, has a curved surface area of 75π cm^2. Find
(a) the value of x,
(b) the volume of the cone, taking $\pi = 3.142$.

23. A heap of rice is conical in shape. The circumference of its base is 8.5 m and its height is 1.2 m. Find the volume of rice, in m^3, correct to 2 significant figures. If the rice is to be stored in bags each of which can hold $\frac{1}{2}$ m^3 of rice, find the number of bags needed. State the assumption that you have about the rice. (Take $\pi = 3.142$)

24. The outer radius of a plastic ball is 4 cm and the inner radius is 3.6 cm. Find the volume of plastic required to make it. (Take $\pi = 3.142$)

Find the volume and surface area of each of the spheres whose individual radius is given below. Take π to be 3.142 and give your answers correct to the nearest whole number.

25. 5.8 cm **26.** 12.6 cm **27.** 24.2 mm **28.** 6.25 m

Using your calculator value for π, find the radius and volume of a sphere with a surface area of

29. 154 cm^2, **30.** 616 mm^2, **31.** 1386 m^2,
32. 113 m^2, **33.** 3850 cm^2.

Taking π to be 3.142, find the radius and surface area of the following spheres, whose volumes are

34. 34 cm^3, **35.** 112 mm^3, **36.** 5276 cm^3, **37.** 68.2 m^3,

correct to one decimal place.

A hemispherical solid has a radius r cm, total surface area A cm^2 and volume V cm^3.
Taking π to be 3.142, find the value of r and of V, correct to one decimal place, given that the value of A is

38. 166.2, **39.** 374, **40.** 74, **41.** 1058.4.

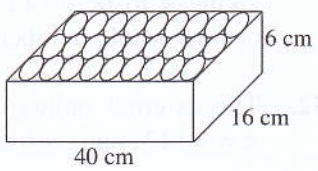
42. Find the mass of a garnium hemisphere of diameter 30 cm if 1 cm^3 of garnium weighs 9.2 g. Take π to be 3.142 and give your answer correct to the nearest kg.

43. Calculate the cost, to the nearest dollar, of painting a sphere of radius 8 m if 1 litre of the paint which costs \$8.50 covers only 8 m^2. (Take π to be 3.142)

44. A mooncake is in cylindrical form of radius 4 cm and height 3 cm. Some mooncakes of this size are packed into a closed rectangular box measuring 40 cm by 16 cm by 6 cm.

(a) Calculate the volume of one mooncake.
(b) At most how many such mooncakes can be packed into the box?
(c) When the box is filled with the mooncakes, find the volume of the empty space in the box.
(d) Can a new box measuring 32 cm by 10 cm by 12 cm hold as many mooncakes as the original box?

(Take $\pi = 3.142$)

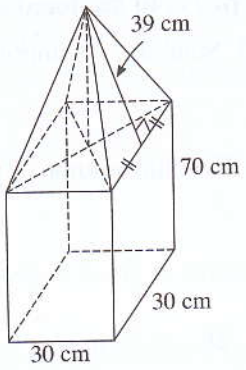


45. A sphere with a diameter of $(x + 2)$ cm has a volume of 972π cm^3. Find

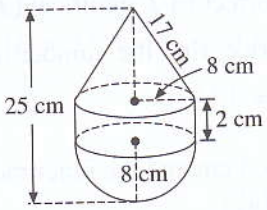
(a) the value of x,
(b) the surface area of the sphere, taking $\pi = 3.142$.

Find the volume and the total surface area of the following solids. (Take π to be 3.142)

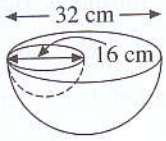
46.



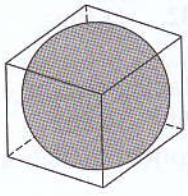
47.



48.



49. A solid sphere of diameter 6 cm fits snugly inside a hollow cube of sides 6 cm each. The cube is closed at the bottom.

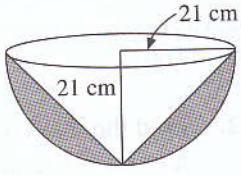


(a) Water is poured into the cube containing the sphere. Calculate the volume of water, in cm^3, needed to completely fill the cube correct to the nearest cm^3.

(b) Express the volume of the sphere as a percentage of the volume of the cube correct to the nearest 0.1%.

(Take $\pi = 3.142$)

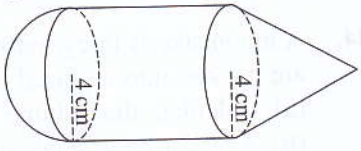
50. A container in the form of a hemisphere has a conical part removed as shown in the diagram.



(a) Find the capacity of the container in litres, using your calculator value of π.

(b) The container is made of material, 1 cm^3 of which weighs 1.5 g. Find the mass of the container correct to the nearest kg.

51. The diagram shows a solid consisting of a cone, a cylinder and a hemisphere. The base radius is 4 cm. The ratio of the volumes of the cone, the cylinder and the sphere is 6 : 27 : 4. Find



(a) the height of the cone,

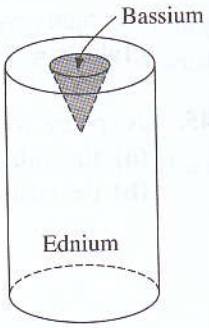
(b) the height of the cylinder,

(c) the volume of the solid,

taking π to be 3.142 and giving your answer correct to the nearest whole number.

52. The external radius of a spherical object is 8 cm and the internal radius is 7.2 cm. Taking $\pi = 3.142$, find, correct to 2 decimal places, the volume of material required to make it.

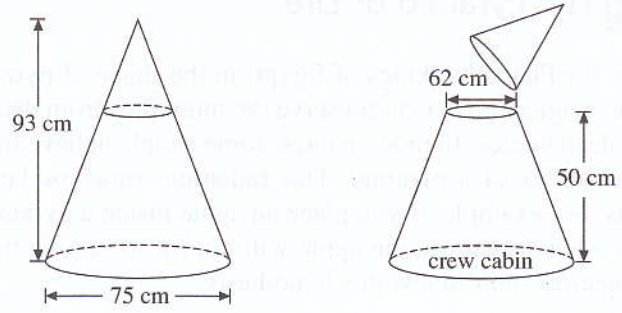
53. A metal cylinder has a radius 2 cm and a height of 3 cm. When manufactured, it should have a certain mass, but when made of ednium metal it is found to be too heavy. To reduce its mass, a conical hole is cut in the metal (as shown in the diagram) and this is completely filled with a lighter metal bassium. The conical hole has a radius of 1 cm and a depth of 0.5 cm. Calculate, correct to three significant figures,

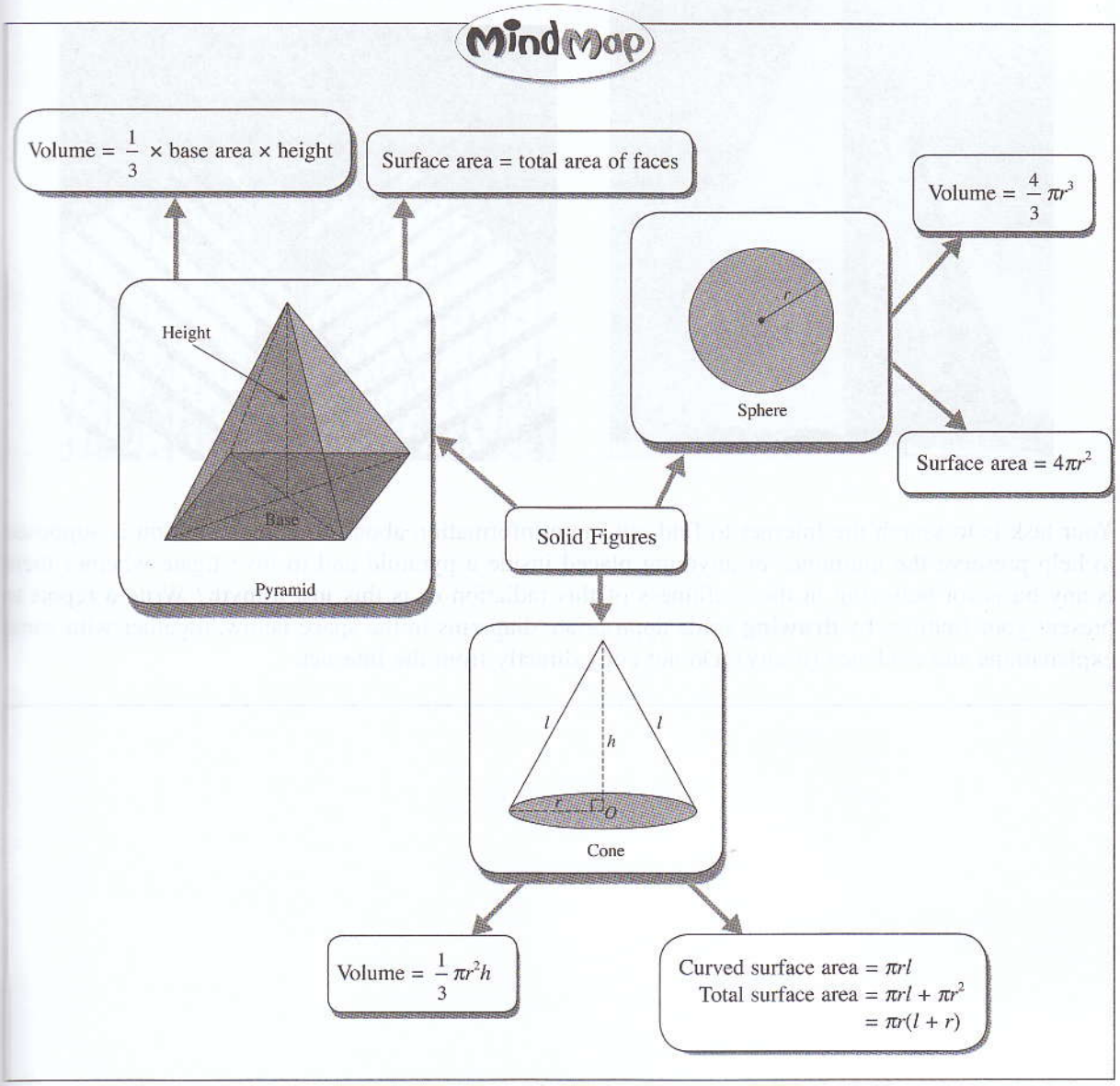


(a) the volume of ednium and the volume of bassium in the cylinder,

(b) the percentage reduction in mass if the density of bassium is half that of ednium. (Take $\pi = 3.142$)

54. The protective nose cone of a re-entry space capsule can be ejected leaving a crew cabin. The diagram shows a model of the space capsule with the crew cabin having a vertical height of 50 cm and a top diameter of 62 cm. Find the volume of the crew cabin of the model. (Take $\pi = 3.142$)

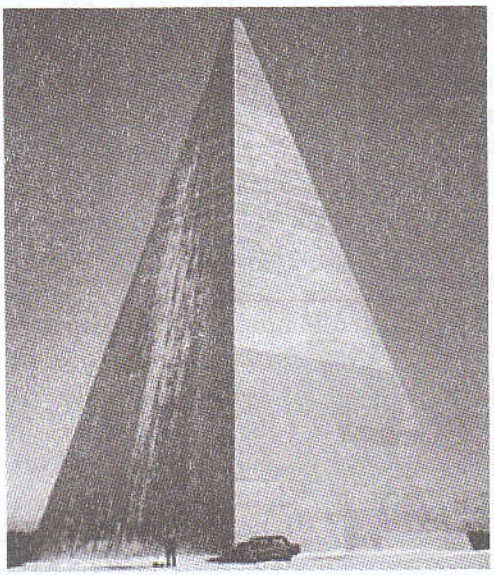




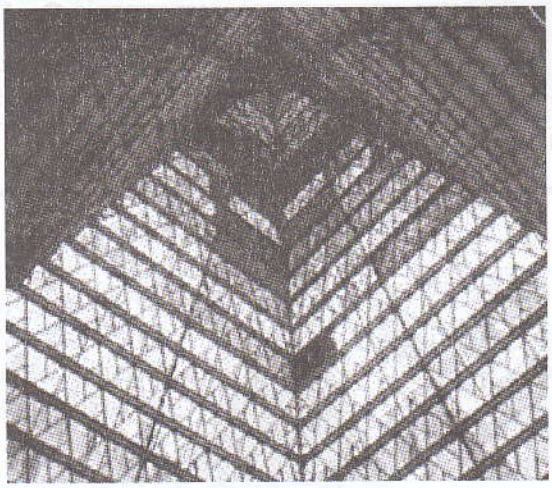
Alternative ASSESSMENT

Performance Task: The Pyramid of Life

The Egyptians built tombs for Pharaohs (kings of Egypt) in the shape of pyramids. They believed that pyramidal tombs had some magical powers to preserve the mummies from decay, other than the special herbs used to embalm the dead bodies. In modern days, some people believe the existence of an ionized column radiating from the vertex of a pyramid. This radiation somehow helps to preserve anything placed inside the pyramids. For example, if you place an apple inside a pyramidal tent built in an open field that is unblocked by other structures, the apple will not rot for a long time. The diagrams below show some pyramidal structures built to test this hypothesis.



Inside the pyramid



Your task is to search the Internet to find out more information about how this radiation is supposed to help preserve the mummies or anything placed inside a pyramid and to investigate whether there is any basis for believing in the usefulness of this radiation or is this just a myth? Write a report to present your findings by **drawing** some appropriate diagrams in the space below, together with some explanations and evidence (if any). Do not copy directly from the Internet.

Scoring Rubric:

Competency Level	Mathematical Concept	Mathematical Communication	Effort
4	Showed complete understanding of pyramids	Gave clear and complete explanations and used accurate mathematical terminology	A thorough research done with lots of examples provided
3	Showed nearly complete understanding of pyramids	Gave nearly complete explanations and/or made some minor errors in mathematical terminology used	Sufficient research done with some good examples provided
2	Showed some understanding of pyramids	Gave incomplete explanation and/or made some errors in mathematical terminology used	Limited research done
1	Showed limited understanding of pyramids	Gave explanations which were difficult to understand and/or made major errors in mathematical terminology used	Research done was unclear or not relevant
0	Showed no understanding of pyramids	Gave muddled explanations and did not use any accurate mathematical terminology	No evidence of research done
Score			

Final Score:

_____ / 12

Final Score	10–12	8–9	6–7	4–5	0–3
Grade	A	B	C	D	F

Teacher's Comments (if any):

Chapter 8 Graphs of Linear Equations in Two Unknowns

Summary

1. A graph is a drawing which shows the relationship between numbers or quantities.

2. Graphs of linear equations are straight lines.

3.

Equation	Graph
$y = c$	Parallel to the x-axis and the gradient is 0
$x = a$	Parallel to the y-axis
$y = mx$	Passes through the origin and has the gradient, m
$y = mx + c$	Cuts the y-axis at the point $(0, c)$ and has the gradient, m

4. The solution of simultaneous linear equations lies at the point of intersection of their graphs.

5. Simultaneous linear equations have an infinite number of solutions if their graphs drawn on the same rectangular plane are identical.

6. Simultaneous linear equations have no solution if their graphs drawn on the same rectangular plane are parallel.

Practice Questions

1. **(a)** Given the equation $3x + 2y = 6$, copy and complete the table below.

x	0	2	4
y			

(b) Draw the graph of $3x + 2y = 6$.

2. (a) Given the equation $y = 2x + 5$, copy and complete the table below.

x	0	1	2
y			

(b) Draw the graph of $y = 2x + 5$.
(c) From the graph, find
(i) the values of y when $x = -0.5$, 0.7 and 1.8,
(ii) the values of x when $y = -3$, 0.6, and 3.

Draw the graphs of the following equations.

3. $y = -4$
4. $y = \frac{1}{2}$
5. $y = -1\frac{1}{2}$
6. $y = 0$
7. $x = 2$
8. $x = 0$
9. $y = -2x$
10. $y = \frac{1}{2}x + 2$
11. $x + 2y = 4$
12. $2x - 3y = 6$
13. $5x + 2y = 10$
14. $3x - 4y = 0$
15. $4x + 3y = 12$

16. The line $x = 4$, $y = x - 2$ and $y = -2$ form the sides of a triangle.
(a) Draw the triangle on graph paper.
(b) Calculate the numerical value of the area of the triangle.

17. The lines $y = 0$, $y = 2$, $y = x$ and $x + y = 6$ form the sides of a trapezium.
(a) Draw the trapezium on graph paper.
(b) Calculate the numerical value of the area of the trapezium.

18. The lines $x = 3$, $y = -\frac{1}{2}(x + 1)$ and $y = \frac{2}{3}x + 3$ form the sides of a triangle.
(a) Draw the triangle on graph paper.
(b) Calculate the numerical value of its area.

Solve the following simultaneous equations using the graphical method.

19. $y = x + 2$
$y = -2x + 2$
20. $2x - y = 3$
$x + y = 0$
21. $3x - y = -1$
$x + y = -3$
22. $7x - 3y = 6$
$7x - 4y = 8$
23. $8x + 3y = 7$
$2x + y = 2$
24. $4x - 7y = 23$
$6x + 2y = -3$
25. $3x + 7y = 17$
$3x - 6y = 4$
26. $3x - 5y = 13$
$5y + x = 7$
27. $2x + \frac{y}{4} = 6$
$4x - y = 6$

Write down the coordinates of the points of intersection of the graphs of the following equations with the y-axis.

28. $y = 7x$
29. $y = -2x$
30. $y = 3x + 5$
31. $y = 6x - 7$
32. $y = -\frac{1}{2}x - 4$
33. $y = -5x + \frac{1}{3}$

34. $y = \frac{1}{3}x - 4$ **35.** $y = \frac{2}{3}x + \frac{1}{5}$ **36.** $2x + y = 0$

37. $3x - 2y = 4$ **38.** $4x + y = -3$ **39.** $-2x - 3y = 6$

Write down the equation of the line cutting the y-axis at the given point and parallel to the given line.

40. $(0, 2)$, $y = 2x + 6$ **41.** $(0, 5)$, $y = -\frac{1}{2}x + 3$

42. $(0, -4)$, $y = -3x + 7$ **43.** $\left(0, -2\frac{1}{2}\right)$, $2x - 3y = -\frac{1}{2}$

44. $(0, -1)$, $y = 9x - 4$ **45.** $(0, 2.6)$, $y = 4x + 5$

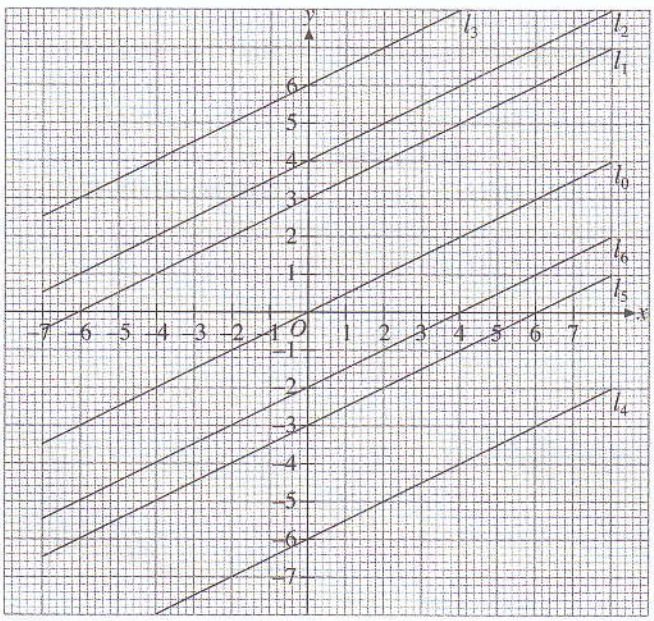
46. $\left(0, -\frac{4}{5}\right)$, $4x + y = -2$ **47.** $(0, 8)$, $x - 5y = 3$

48. $(0, -7)$, $2x + 3y = 1$

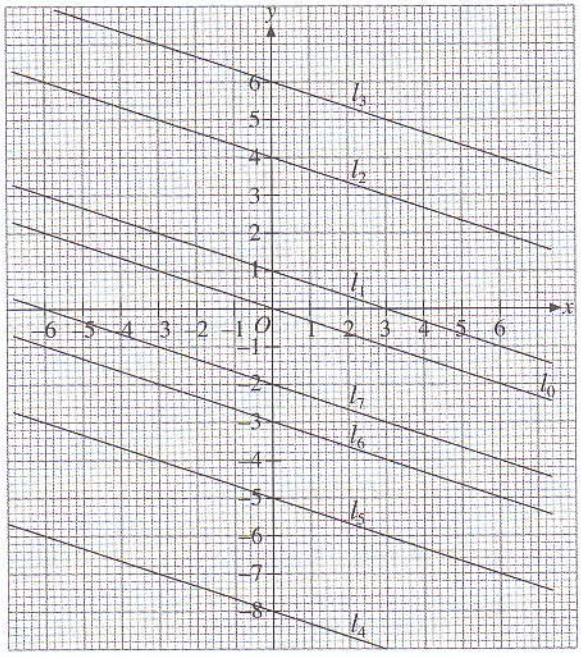
49. Find the coordinates of the point at which the line $\frac{x}{3} + \frac{y}{2} = 1$ cuts the y-axis.

50. The straight line $y = mx + c$ is parallel to the straight line $y = 4x + 3$ and passes through the point $\left(0, -\frac{1}{2}\right)$. Write down the values of m and c.

51. Given that the equation of the line l_0 is $y = \frac{1}{2}x$, write down the equations of the lines l_1, l_2, l_3, l_4, l_5 and l_6.



52. Given that the equation of the line l_1 is $y = -\frac{1}{3}x + 1$, write down the equations of the lines l_0, l_2, l_3, l_4, l_5, l_6 and l_7.



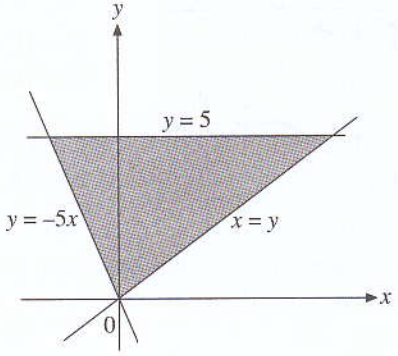
53. The equation of a straight line is $3x + 2y = 5$. Given that the point $(k, 4)$ lies on the line, find the value of k.

54. A straight line L passes through the point $(0, -1)$ and is parallel to the straight line $y = 3x + 1$.
(a) Write down the equation of the straight line L.
(b) Given that L passes through the point $(2, b)$, find the value of b.

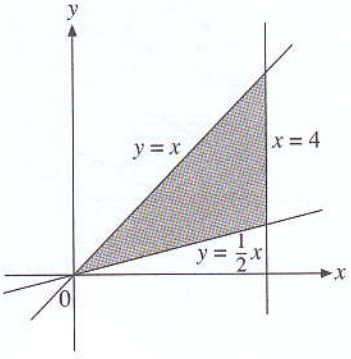
55. The equation of the line l is $3y - x = 9$. Find
(a) the coordinates of the point where l intersects the y-axis,
(b) the equation of the line which is parallel to l and which passes through the point $(0, -5)$.

56. Find the area of the shaded region in the diagram.

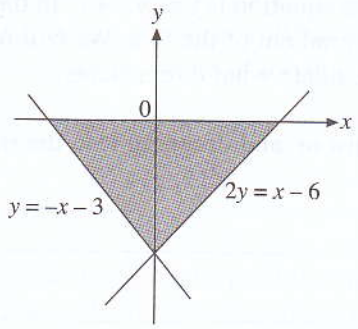
(a)



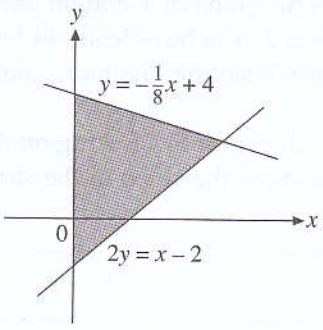
(b)



(c)



(d)



57. **(a)** Explain why the simultaneous equations $6x - 3y = 4$ and $y = 2x + 5$ have no solution. What can you say about the straight lines representing these two equations?

(b) Explain why the simultaneous equations $6y + 3x = 15$ and $y = -\frac{1}{2}x + \frac{5}{2}$ have an infinite number of solutions. What can you say about the two straight lines representing the equations?

Alternative ASSESSMENT

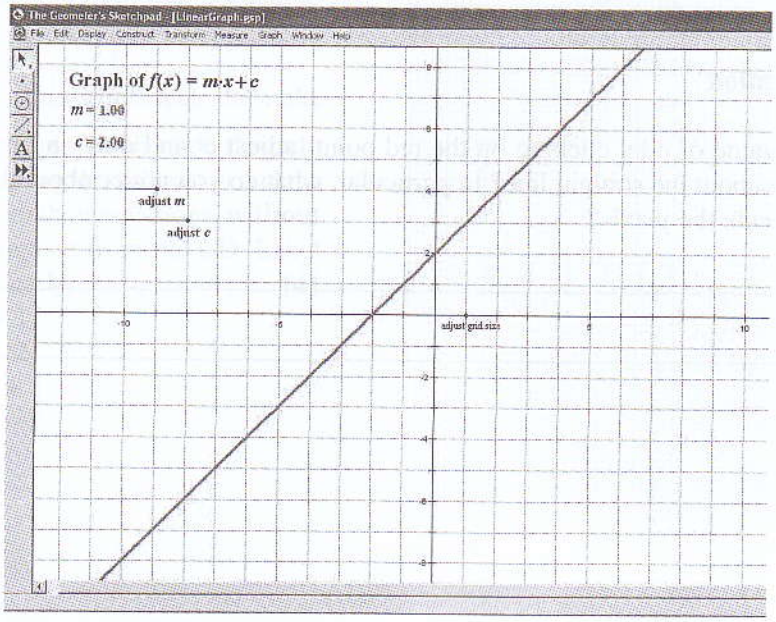
IT Worksheet: Equation of a Straight Line

You need the Geometer's Sketchpad (GSP), a dynamic geometry software, to view and interact with the GSP template for this worksheet. If your school does not have a licensed copy of version 4, you may download the free evaluation version from www.keypress.com for trial first.

The purpose of this worksheet is to investigate how the gradient and y-intercept of a straight line affect what its graph looks like, and how these determine the form of the equation of the line.

Section A: Revision of Sec 1 Topic

Open the appropriate template from the Workbook CD.



The template shows the graph of a straight line (pink line) whose equation is $y = mx + c$. In the template shown, $m = 1$ and $c = 2$. You have learnt in Sec 1 that m is the gradient of the line. We will revise how the gradient m affects what the line looks like, before we investigate what c represents.

1. Increase the value of m by clicking on the red point 'adjust m' and dragging it to the right. What do you notice about the slope of the straight line? [1]

2. Decrease the value of m by clicking on the red point 'adjust m' and dragging it to the left until the value of m is **negative**. What do you notice about the slope of the straight line? [1]

3. Keep the value of m **negative**. Decrease the value of m further by clicking on the red point 'adjust m' and dragging it to the left. Does the straight line become steeper or less steep? Does the gradient of the line (i.e. the value of m) increase or decrease? [2]

4. Can you conclude that if the gradient m of a straight line decreases, then the line will become less steep? Why or why not? [2]

Section B: Exploration

5. Increase the value of c by clicking on the red point 'adjust c' and dragging it to the right. What do you notice about the straight line? In particular, what do you notice about the slope of the line and where it cuts the y-axis? [3]

6. Decrease the value of c by clicking on the red point 'adjust c' and dragging it to the left until the value of c is negative. What do you notice about the straight line? In particular, what do you notice about the slope of the line and where it cuts the y-axis when c is negative? **[3]**

7. c is called the y-intercept of the straight line whose equation is $y = m\,x + c$. Can you explain why the line will cut the y-axis at the point $(0, c)$? **[2]**

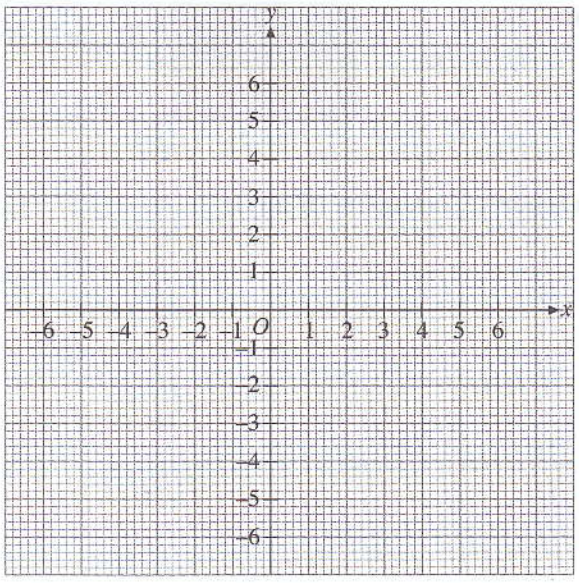
Section C: Application

8. For each equation of the straight line, $y = m\,x + c$, given below, write down its gradient m and y-intercept c. **[4]**

Equation of Straight Line $y = mx + c$	Gradient of Line, m	y-intercept, c
$y = 2x + 1$		
$y = x - 3$		
$y = -x + 4$		
$y = -5x$		

9. You have learnt in Sec 1 that the gradient m of a straight line is equal to $\frac{\text{rise}}{\text{run}}$. Applying what you have learnt in Sec 1 and in this worksheet, draw the following graphs, in the same diagram on the right, using the value of m and the value of c from their equations, **without** using the template from the CD. Label the graphs clearly. **[4]**

(a) $y = 2x + 1$
(b) $y = 2x - 2$
(c) $y = -2x + 1$
(d) $y = -3x - 2$



10. With reference to what you have drawn in Q9, what do the graphs of $y = 2x + 1$ and $y = 2x - 2$ have in common? What is the special name used to describe this common feature? **[2]**

_____

_____

11. With reference to what you have drawn in Q9, what do the graphs of $y = 2x - 2$ and $y = -3x - 2$ have in common? **[1]**

_____

_____

12. With reference to what you have drawn in Q9, how can you obtain the graph of $y = -2x + 1$ from the graph of $y = 2x + 1$? **[1]**

_____

_____

Section D: Conclusion

13. Write down one main lesson that you have learnt from this worksheet. **[1]**

_____

_____

Final Score:

_____ / 25

Final Score	20–25	16–19	13–15	9–12	0–8
Grade	A	B	C	D	F

Teacher's Comments (if any):

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_____

Chapter 9 Graphs of Quadratic Functions

Summary

1. The general form of a quadratic graph is $y = ax^2 + bx + c$ $(a \neq 0)$.

2. The quadratic graph of $y = ax^2 + bx + c$ $(a \neq 0)$ has a minimum point (the lowest point) when a is positive. It has a maximum point (the highest point) when a is negative.

3. The line of symmetry of the quadratic graph passes through the maximum or minimum point.

Practice Questions

1. **(a)** Given that $y = x^2 + 1$, copy and complete the following table.

x	–3	–2	–1	0	1	2	3	4
y	10			1	2			17

(b) Taking 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = x^2 + 1$ from $x = -3$ to $x = 4$.
(c) Find the equation of the line of symmetry.
(d) Write down the coordinates of the minimum point.

2. **(a)** Given that $y = x^2 - 4x$, copy and complete the following table.

x	–2	–1	0	1	2	3	4
y	12		0				0

(b) Taking 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = x^2 - 4x$ from $x = -2$ to $x = 4$.
(c) Find the equation of the line of symmetry.
(d) Write down the coordinates of the minimum value of y.

3. Given that $y = 6x - x^2$, copy and complete the following table.

x	0	1	2	3	4	5	6
y	0	5			8		0

(a) Using a scale of 2 cm to 1 unit on each axis, draw the graph of $y = 6x - x^2$ for values of x in the range $0 \leq x \leq 6$.
(b) Write down the equation of the axis of symmetry of the graph.
(c) Write down the coordinates of the turning point and state whether it is minimum or maximum.

4. The variables x and y are connected by the equation $y = x^2 - 5x + 5$.
Some corresponding values of x and y are given in the following table.

x	0	1	2	3	4	5
y	5	a	–1	b	c	5

(a) Calculate the values of a, b and c.
(b) Taking 2 cm to represent 1 unit on each axis, draw the graph of $y = x^2 - 5x + 5$ for values of x in the range of $0 \leq x \leq 5$.
(c) (i) Find the x-coordinates of the points on the graph where $y = 2$.
(ii) Find the y-coordinates of the point on the graph where $x = 4.5$.

5. The variables x and y are connected by the equation $y = (x + 3)(x - 2)$ and some corresponding values are given in the table below.

x	–4	–3	–2	–1	0	1	2	3
y	6	0	a	b	–6	–4	0	6

(a) Calculate the values of a and b.
(b) Taking 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = (x + 3)(x - 2)$ for values of x in the range $-4 \leq x \leq 3$.
(c) Find, from your graph,
(i) the value of y when $x = 2.6$,
(ii) the values of x when $y = 1$.

6. The following is an incomplete table of values for the graph of $y = 3 + 13x - 4x^2$.

x	–2	–1	0	1	2	3	4	5
y	–39	–14	3		13		–9	

(a) Calculate the missing values of y.
(b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 10 units on the y-axis, draw the graph of $y = 3 + 13x - 4x^2$ for the range $-2 \leq x \leq 5$.
(c) Find, from your graph, the values of x when
(i) $y = 0$, (ii) $y = -10$.

7. The following is an incomplete table of values for the graph of $y = x^2 + 2x - 8$.

x	–5	–4	–3	–2	–1	0	1	2	3
y	7		–5			–8	–5		7

(a) Calculate the missing values of y.
(b) Taking a scale of 2 cm to 1 unit on the x-axis and 1 cm to 1 unit on the y-axis, draw the graph of $y = x^2 + 2x - 8$ for the range $-5 \leq x \leq 3$.
(c) Find, from your graph, the values of x when
(i) $y = 0$, **(ii)** $y = -2$, **(iii)** $y = \frac{1}{2}$.
(d) Write down the equation of the line of symmetry and the least value of y.

8. The variables u and v are connected by the equation $u = 10 - v - v^2$. Some corresponding values of u and v are given in the following table.

v	–4	–3	–2	–1	0	1	2	3
u	–2	a	8	b	10	8	c	–2

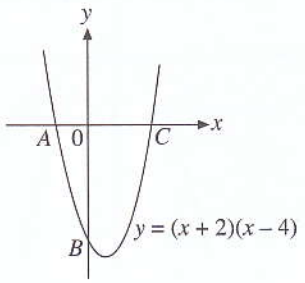
(a) Calculate the values of a, b and c.
(b) Taking 2 cm to represent 1 cm on the v-axis and 1 cm to represent 1 unit on the u-axis, draw the graph of $u = 10 - v - v^2$ for values of v from –4 to 3.
(c) From your graph, find the values of
(i) u when $v = -2.5$, 0.6 and 2.2, **(ii)** v when $u = 0$, 5 and 7.5,
(d) Write down the equation of the line of symmetry and the coordinates of the points where u has a maximum value.

9. **(a)** Copy and complete the table of values for $y = 2x^2 - 3x - 7$ given below.

x	–2	–1	0	0.5	1	2	3	4
y			–7		–8		2	13

(b) Using 2 cm along the x-axis to represent 1 unit and 2 cm along the y-axis to represent 5 units, draw the graph of $y = 2x^2 - 3x - 7$ and use it to answer the questions in **(c)** and **(d)**.
(c) Find the value of y when $x = 3.6$.
(d) Find the values of x when $y = 2$.

10. The curve $y = (x + 2)(x - 4)$ cuts the x-axis at the points A and C, and the y-axis at B.
(a) Write down the coordinates of the points A, B and C.
(b) Find the equation of the line of symmetry of the curve. Hence find the coordinates of the minimum point.



Draw the following graphs using the axes indicated.

11. $y = 3x^2$; x from -3 to 3.
Scales: x-axis 2 cm = 1 unit
y-axis 2 cm = 5 units

12. $y = (x - 1)(5 - x)$; x from 0 to 5
Scales: x-axis 2 cm = 1 unit
y-axis 2 cm = 1 unit

13. $y = 3 - 2x - x^2$; x from -4 to 2.
Scales: x-axis 2 cm = 1 unit
y-axis 2 cm = 1 unit

14. $y = \frac{x^2}{2} - 2x$; x from -1 to 6
Scales: x-axis 2 cm = 1 unit
y-axis 2 cm = 1 unit

15. $y = 6 + 2x - x^2$; x from -3 to 4.
Scales: x-axis 2 cm = 1 unit
y-axis 2 cm = 5 units

16. $y = \frac{1}{2}(6 + x - 2x^2)$; x from -2 to 3
Scales: x-axis 2 cm = 1 unit
y-axis 2 cm = 1 unit

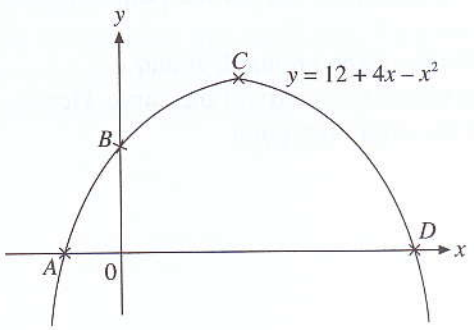
17. Taking 2 cm to represent 1 unit on each axis, draw the graph of $y = 7x - 2x^2 - 4$ for values of x from 0 to $3\frac{1}{2}$. From your graph,
(a) find
(i) the values of y when $x = 0.8$ and 1.7,
(ii) the values of x when $y = -1$ and 2.
(b) the greatest value of y and the value of x which gives this value of y.

18. Taking 2 cm to represent 1 unit on each axis, draw the graph of $y = x^2 - x - 2$ by plotting the eight points for which $x = -2, -1\frac{1}{2}, -1, 0, \frac{1}{2}, 1, 2$ and 3. From your graph, find
(a) the values of x for which $y = 1$,
(b) the value of x for which gives the least value of y.

19. Using suitable scales, plot the graph of $y = x^2 - 4x - 3$ (take values of x from -2 to 6). Use your graph to find
(a) y when $x = 2.5$, **(b)** x when $y = 6$.

20. Using suitable scales, plot the graph of $y = 2x^2 + 4x - 7$ (take values of x from -5 to 3). Use your graph to find
(a) y when $x = -4.2$, **(b)** x when $y = 14$.

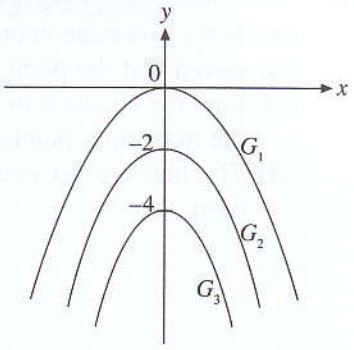
21. In the sketch, the curve $y = 12 + 4x - x^2$ cuts the x-axis at two points A and D, and the y-axis at B. Given that C is the maximum point,
(a) write down the coordinates of the points A, B, C and D,
(b) find the equation of the line of symmetry of the curve.



22. In the sketch, the equations of two of the graphs are given by $y = -x^2$ and $y = -x^2 - 2$.

(a) Identify the two graphs that represent the two given equations.

(b) Hence write down the equation of graph G_3. State the equation of the line of symmetry of G_3 and its maximum point.



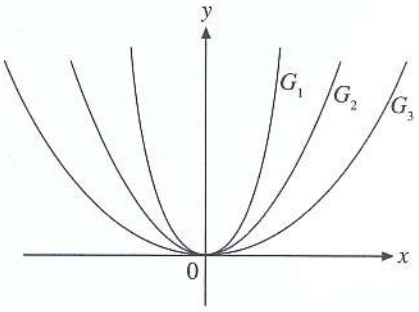
23. The diagram shows the graphs of G_1: $y = ax^2$, G_2: $y = bx^2$, G_3: $y = cx^2$. Identify which of the following cases is true.

Case I: $a > b > c$

Case II: $a < b < c$

Case III: $c > b$, $a > b$

Case IV: $c < b$, $a < b$

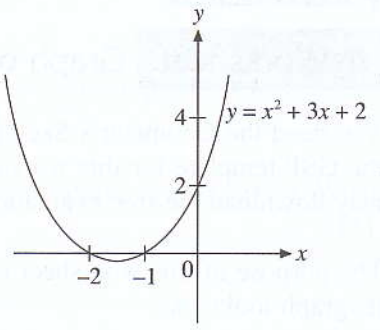


24. The sketch shows the graph of $y = x^2 + 3x + 2$.

(a) Write down the solutions of the equation $x^2 + 3x + 2 = 0$.

(b) Find the equation of the line of symmetry.

(c) State also the y-intercept.



25. The diagram shows the graphs of $y = ax^2$ and $y = bx^2$. Which of the following statements is true?

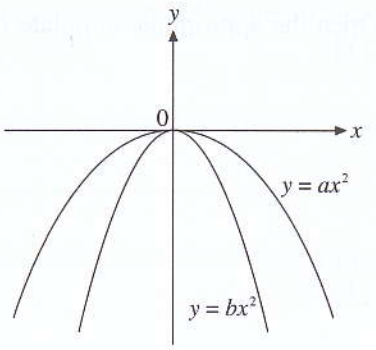
(A) $a > 0$, $b > 0$, $a > b$

(B) $a > 0$, $b < 0$, $a < b$

(C) $a > 0$, $b > 0$, $a < b$

(D) $a < 0$, $b < 0$, $a > b$

(E) $a < 0$, $b < 0$, $a < b$



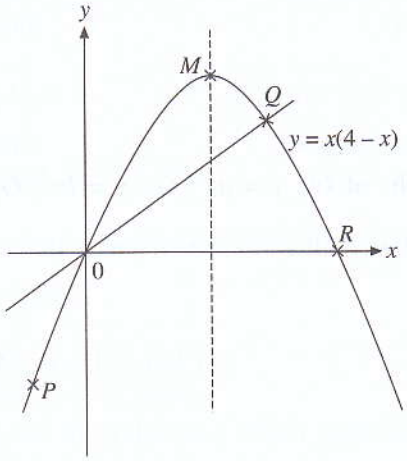
26. The curve $y = x(4 - x)$ cuts the x-axis at the origin and at R.

(a) Write down the coordinates of R.

(b) Given that the point $P(-1, k)$ lies on the curve, find the value of k.

(c) Find the equation of the line of symmetry of the curve. Hence write down the coordinates of the maximum point M.

(d) The line $y = mx$ cuts the curve at the origin and at point $Q(3, p)$. Find the value of p and of m.



Alternative ASSESSMENT

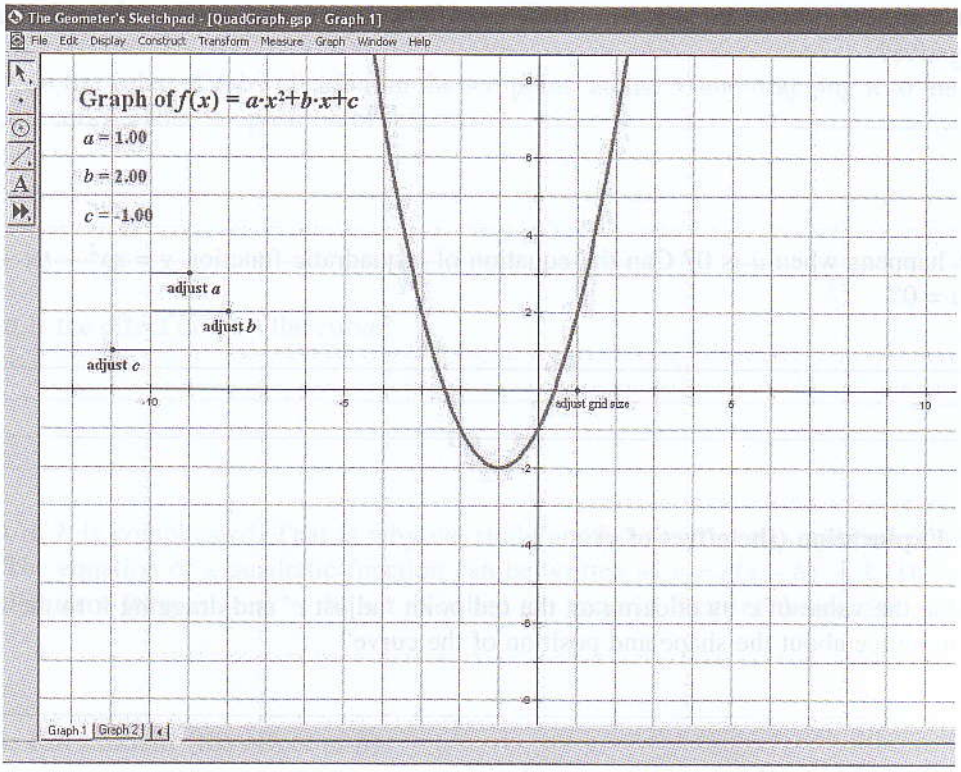
IT Worksheet: Graph of a Quadratic Function

You need the Geometer's Sketchpad (GSP), a dynamic geometry software, to view and interact with the GSP template for this worksheet. If your school does not have a licensed copy of version 4, you may download the free evaluation version from www.keypress.com for trial first.

The purpose of this worksheet is to investigate how the equation of a quadratic function affects what its graph looks like.

Section A: Exploration (the effect of a)

Open the appropriate template from the Workbook CD.



The template shows the graph of a quadratic function (pink curve) whose equation is $y = ax^2 + bx + c$. In the template shown above, $a = 1$, $b = 2$ and $c = -1$.

1. Adjust the value of b to **0** by clicking on the red point 'adjust b' and dragging it to the left. Similarly, adjust the value of c to **0**.

2. Increase the value of a by clicking on the red point 'adjust a' and dragging it to the right. What do you notice about the shape of the curve? **[1]**

3. Keep the value of a **positive**. Decrease the value of a by clicking on the red point 'adjust a' and dragging it to the left. What do you notice about the shape of the curve? **[1]**

4. Decrease the value of a further until it is **negative**. What do you notice about the shape of the curve? **[1]**

5. What is the effect of a on the shape of the curve? What happens when a is positive and when a is negative? [2]

6. What happens when a is 0? Can the equation of a quadratic function $y = ax^2 + bx + c$ be such that $a = 0$? [2]

Section B: Exploration (the effect of c)

7. Increase the value of c by clicking on the red point 'adjust c' and dragging it to the right. What do you notice about the shape and position of the curve? [2]

8. Decrease the value of c by clicking on the red point 'adjust c' and dragging it to the left until it is negative. What do you notice about the position of the curve? [1]

9. What is the effect of c on the curve? [1]

Section C: Exploration (the effect of b)

10. Adjust the value of a to **1**, the value of b to **−2** and the value of c to **0**.

11. Increase the value of b by clicking on the red point 'adjust b' and dragging it to the right. What do you notice about the shape and position of the curve? [2]

12. Adjust the value of a to -1, the value of b to -2 and the value of c to 0.

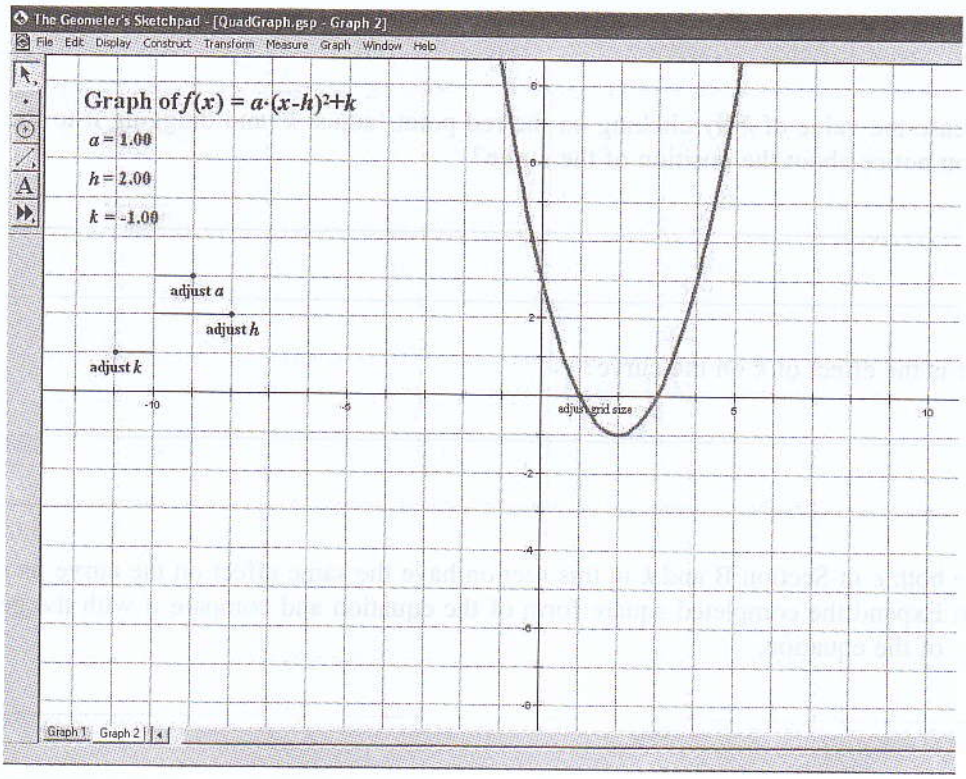
13. Increase the value of b by clicking on the red point 'adjust b' and dragging it to the right. What do you notice about the position of the curve? [1]

14. What is the effect of b on the curve? [1]

The effect of b is complicated. That is why we study another form of the equation of the quadratic function. The equation of a quadratic function can be written as $y = a(x - h)^2 + k$. This is called the **completed-square form**. $y = ax^2 + bx + c$ is called the **general form** of the equation of a quadratic function.

Section D: Enrichment (the effect of h)

At the bottom left corner of the template, click on the tab 'Graph 2'. This will show the template below. The equation of the curve is $y = a(x - h)^2 + k$.



15. Increase the value of h by clicking on the red point 'adjust h' and dragging it to the right. What do you notice about the position of the curve? [1]

16. Decrease the value of h by clicking on the red point 'adjust h' and dragging it to the left. What do you notice about the position of the curve? [1]

17. What is the effect of h on the curve? [1]

Section E: Enrichment (the effect of k)

18. Increase the value of k by clicking on the red point 'adjust k' and dragging it to the right. What do you notice about the position of the curve? [1]

19. Decrease the value of k by clicking on the red point 'adjust k' and dragging it to the left. What do you notice about the position of the curve? [1]

20. What is the effect of k on the curve? [1]

21. Since both c in Section B and k in this section have the same effect on the curve, is $c = k$? [1]
Hint: Expand the completed-square form of the equation and compare it with the general form of the equation.

22. Express b in Section C in terms of a and h of the completed-square form of the equation. Can you explain why changing b will move the curve in the complicated way described in Q14? **[2]**
Hint: Use Q21 also.

Section F: Conclusion

23. Write down one main lesson that you have learnt from this worksheet. **[1]**

Final Score: ☐ / 22

Final Score	18–22	14–17	11–13	8–10	0–7
Grade	A	B	C	D	F

Teacher's Comments (if any):

Term III Revision Test Time: $1\frac{1}{2}$ h

1. Using a scale of 2 cm to represent 1 unit on both the x- and y-axes, draw the graph of $2x + 3y + 5 = 0$ and $3x + 2y = 0$ and hence solve the equations for y and x by the graphical method. **[5]**

2. Given that $y = 5 - x - x^2$, copy and complete the following table.

x	–3	–2	–1	0	1	2
y	–1			5	3	

Using a scale of 2 cm to represent 1 unit on both axes, draw the graph of $y = 5 - x - x^2$ for $-3 \le x \le 2$. Use your graph to find

(a) the equation of the line of symmetry,
(b) the greatest value of y,
(c) the value of y for which $x = -1.6$,
(d) the values of x for which $y = 4$. **[7]**

3. Given that $y = 3x^2 - 4x - 30$, copy and complete the following table.

x	–3	–2	–1	0	1	2	3	4	5
y	9		–23	–30			–15		25

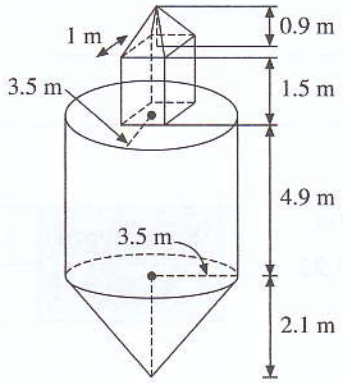
Plot the graph of $y = 3x^2 - 4x - 30$ using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 10 units on the y-axis. Use your graph to find

(a) the value of y when $x = 3.6$,
(b) the values of x when $3x^2 - 4x - 30 = 0$,
(c) the values of x when $3x^2 - 40 - 30 = -20$. **[7]**

4. Gigia Chemicals Pte Ltd has a storage tank made up of a regular pyramid with a square base of sides 1 m each and height 0.9 m, a cuboid of height 1.5 m, a cylinder of radius 3.5 m and height 4.9 m and a cone of height 2.1 m as shown in the diagram.

(a) Find the volume of the storage tank
(i) in m^3,
(ii) in cm^3, giving your answer in standard form correct to two significant figures.

(b) If a similar tank is made with the radius of the cylinder equal to $2\frac{1}{3}$ m, what will be the volume in m^3, of this new tank? Give your answer correct to one decimal place. **[8]**



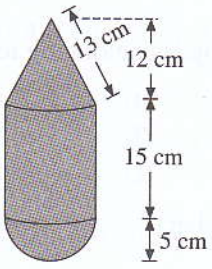
5. Using a scale of 2 cm to represent 1 unit on both the x- and y-axes, draw the graphs of $y = \frac{1}{2}x + 1$ and $x + y = 4$. Hence, determine the coordinates of the point of intersection of the two graphs. **[5]**

6. The following is an incomplete table of values for the graph of $s = 6 - \frac{1}{2}t^2$.

s	–4	–3	–2	–1	0	1	3	4
t			4	$5\frac{1}{2}$	6	$5\frac{1}{2}$		

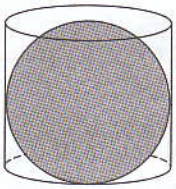
(a) Calculate the missing values of t.
(b) Using a scale of 2 cm to represent 1 unit on each axis, draw the graph of $s = 6 - \frac{1}{2}t^2$ for values of t from –4 to 4 inclusive.
(c) Use your graph to find
(i) the equation of the line of symmetry,
(ii) the value of s when $t = -2.2$,
(iii) the value of t when $s = 2$. **[7]**

7. **(a)** The diagram shows a solid made up of a cone, a cylinder and a hemisphere. Calculate
   (i) the volume, in cm^3, of the solid correct to the nearest whole number,
   (ii) the cost of plating it with material costing \$1.40 per cm^2.
   (Take π to be 3.142)



   (b) A metallic sphere of radius $13\frac{1}{2}$ cm is melted down and recast into small cones of radius $4\frac{1}{2}$ cm and height 6 cm each. Find the number of cones that can be made. **[8]**

8. A sphere fits snugly into a cylinder which has the same height and diameter as the sphere. If the diameter of the sphere is 84 mm, find
   (a) the volume of the cylinder in mm^3, giving your answer correct to two significant figures,
   (b) the volume of the sphere in m^3, giving your answer correct to two significant figures,
   (c) the volume of the cylinder which is not occupied correct to the nearest cm^3. **[6]**



9. The variables S and t are connected by the equation $S = 10 - t - t^2$.
   Some corresponding values are given in the following table.

t	−4	−3	−2	−1	0	1	2	3
S	−2	4	8	10	10	p	q	r

   (a) Calculate the values of p, q and r.
   (b) Taking 2 cm to represent 1 unit on the t-axis and 1 cm to represent 1 unit on the S-axis, draw the graph of $S = 10 - t - t^2$ for values of t in the range $-4 \leq t \leq 3$.
   (c) From your graph, estimate
   (i) the greatest value of S,
   (ii) the values of t for which $S = 0$,
   (iii) the positive value of t for which $S = 5$. **[7]**

Chapter 10 Set Language and Notation

Summary

1. A set is a collection of objects which are clearly defined. The objects belonging to a set are called its **elements** or members.

2. A set can be defined by
   (a) listing its element within braces, e.g. {Ahmad, Ali, John},
   (b) stating its characteristics in words, e.g. {letters in the alphabet},
   (c) drawing a Venn diagram.

3. The **empty** or **null** set is the set containing no element. It is denoted by $\varnothing$.

4. Two sets are **equal** if they have exactly the same elements.

5. A is said to be a **subset** of B, written as $A \subseteq B$, if all the elements of A are also the elements of B.

6. $\varnothing$ is a subset of every set.

7. The **complement** of the set A, written as A', is the set of elements in the universal set, usually denoted by ε, which are not members of set A.

8. The **intersection** of set A and set B, written as $A \cap B$, is the set of elements common to both A and B.

9. The **union** of set A and set B, written as $A \cup B$, is the set of elements which are in set A, or set B or in both set A and set B.

Practice Questions

1. List the members of the following sets.
   (a) $A = \{x : x \text{ is an even number and } 4 < x \leq 22\}$
   (b) $B = \{x : x \text{ is a positive integer less than 15 and } x \text{ is not prime}\}$
   (c) $C = \{x : x \text{ is the month of the year with fewer than 30 days}\}$
   (d) $D = \{\text{days of the week beginning with the letter } S\}$
   (e) $E = \{\text{prime numbers less than 11}\}$
   (f) $F = \{\text{months of the year beginning with the letter } J\}$

2. If $A = \{2, 3, 5, 7, 8, 11\}$, state which of the following statements is true (T) and which is false (F).
(a) $2 \in A$ **(b)** $7 \notin A$ **(c)** $\{7, 8\} \subseteq A$ **(d)** $\{4, 5\} \subseteq A$
(e) $A \subseteq \{3, 4, 5\}$ **(f)** $A \subset \{5, 8, 11\}$

3. If $P = \{g, o, h, d, s\}$, state which of the following statements is true (T) and which is false (F).
(a) $g \in P$ **(b)** $goh \in P$ **(c)** $\{g\} \in P$ **(d)** $\{g, o, h\} \in P$
(e) $\{g\} \subseteq P$ **(f)** $\{g, o, d\} \subseteq P$ **(g)** $\{goh\} \subseteq P$ **(h)** $\{g, o\} \subseteq P$

4. Given that $A = \{a, b, \{c\}, k, t\}$, state whether each of the following statements is true (T) or false (F).
(a) $a \in A$ **(b)** $\{a\} \subseteq A$ **(c)** $\{c\} \in A$ **(d)** $\{a, c\} \subseteq A$
(e) $\{b\} \in A$ **(f)** $\{k, t\} \subseteq A$ **(g)** $\{\{c\}, k, t\} \subseteq A$ **(h)** $\varnothing \subseteq A$
(i) $\{\{c\}\} \in A$ **(j)** $\{c\} \subseteq A$

5. State whether each of the following statements is true (T) or false (F).
(a) $a \notin \{b, c, \{a\}\}$
(b) If $A \subseteq B$ and $B \subseteq A$, then $A = B$.
(c) If $P \cup Q = Q$, then $Q \subseteq P$.
(d) If $A \cap B = A \cup B$, then $A = B$.
(e) If $A \cap B = \{a\}$, then $a \in A$ and $a \in B$.
(f) If $A \cup B = \{a, b, c, d\}$, then $a \in A$ and $a \in B$.
(g) $a \in \{a, b, c, d\}$.
(h) $a \subseteq \{a, b, c, d\}$.

6. Given that $\varepsilon = \{\text{prime numbers less than } 50\}$, $A = \{x : 17 < x \leq 41\}$ and $B = \{x : 5 < x \leq 29\}$, list the members of the following.
(a) $A \cup B$ **(b)** $A \cap B$ **(c)** $A' \cup B$ **(d)** $A' \cup B'$

7. Given that $\varepsilon = \{x : x \text{ is an integer and } 10 < x \leq 23\}$, $A = \{x : x \text{ is a prime number}\}$, $B = \{x : x \text{ is divisible by } 3\}$ and $C = \{x : x > 17\}$,
(a) Find
(i) $n(B \cap C)$, **(ii)** $A \cap B$.
(b) Find the elements x such that $x \in (B' \cup C')$ and $x \notin A$.

8. Given that $\varepsilon = \{x : x \text{ is a real number and } -5 \leq x \leq 10\}$, $A = \{x : -2 \leq x \leq 6\}$ and $B = \{x : -3.5 \leq x \leq 3.5\}$, find
(a) $A \cup B$, **(b)** $(A \cap B)'$, **(c)** $A \cap B'$.
Illustrate each of the answers with a number line.

9. Given that $\varepsilon = \{x : x \text{ is a positive even number}\}$, $A = \{x : 0 \leq x \leq 10\}$ and $B = \{x : 1 \leq x^2 < 82\}$, list the elements of
(a) B, **(b)** $A \cap B$.

10. Given that the universal set ε is the set of positive integers, $A = \{x : x \text{ is prime}\}$, $B = \{x : x \text{ is divisible by } 3\}$ and $C = \{x : 20 \leq x \leq 50\}$, list the elements of
(a) $A \cap B$, **(b)** $B \cap C$.

11. Given that $\varepsilon = \{x : x \text{ is a whole number and } 1 \leq x \leq 12\}$, $A = \{2, 3, 6, 8, 10\}$, $A \cap B = \{3, 6, 8\}$ and $A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10, 12\}$, illustrate the above data on a clearly labelled Venn diagram and list the elements of the set B.

12. Given that $\varepsilon = \{\text{polygons}\}$, $A = \{\text{quadrilaterals}\}$ and $B = \{\text{regular polygons}\}$,
(a) name a member of the set $A \cap B$,
(b) name a member of the set $A \cap B'$ whose diagonals bisect each other.

13. Given that $\varepsilon = \{x : x \text{ is an integer, } 12 \leq x \leq 39\}$, $A = \{x : x \text{ is a multiple of } 5\}$, $B = \{x : x \text{ is a perfect square}\}$ and $C = \{x : x \text{ is odd}\}$, list the members of each of the following.
(a) $A \cap B$ **(b)** $A \cap C$ **(c)** $B \cup C$

14. Given that the universal set is the set of integers, $A = \{x : x > 4\}$, $B = \{x : -1 < x \leq 10\}$ and $C = \{x : x < 8\}$, use similar set notation to describe each of the following.
(a) $A \cap B$ **(b)** $B \cap C$ **(c)** $A' \cap B$ **(d)** $A' \cap C$

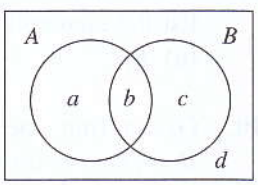
15. Given that $\varepsilon = \{x : x \text{ is a positive integer and } x < 20\}$, $A = \{x : 4 < x < 15\}$ and $B = \{x : 17 < 2x < 37\}$, find
(a) $n(A \cap B)$, **(b)** $n(A \cup B')$, **(c)** $n(A' \cap B)$.

16. Given that $\varepsilon = \{x : x \text{ is an integer, } 0 \leq x < 25\}$, $A = \{x : x \text{ is odd and } 2x > 17\}$, $B = \{x : x \text{ is divisible by } 5\}$ and $C = \{x : x \text{ is prime and } x \leq 19\}$, list the elements of the sets B and C.

17. Given that $\varepsilon = \{x : x \text{ is an integer and } 0 < x \leq 13\}$, $A = \{x : 2x > 9\}$, $B = \{x : (x - 2)(x - 5) = 0\}$ and $C = \{x : x \text{ is prime}\}$,
(a) list the elements of the set
(i) A, **(ii)** B,
(b) list the elements of $A \cap C$,
(c) find the value of $n(A \cap C')$.

18. Fill in the blanks with the symbol, $\in$, $\notin$, $=$, $\subseteq$ or $\supseteq$ so as to make the following statements correct.
(a) $\{3, 5\}$ _____ $\{3, 7, 5, 9\}$
(b) $\{2, 5, 6\}$ _____ $\{6, 5, 2, 6\}$
(c) $(3, 6, 9, 12\}$ _____ {multiples of 3}
(d) $\{5, 6, 7, 8\}$ _____ $\{x : x \text{ is an integer, } 4 \leq x < 9\}$
(e) $\{x : 1 \leq x \leq 9\}$ _____ $\{x : 2 < x \leq 8\}$
(f) *go* _____ {*go*, *goh*, *gosh*}
(g) *god* _____ {*g*, *o*, *d*, *go*}
(h) $\{2, 3, 5, 7\}$ _____ {prime numbers less than 10}

19. Given that $\varepsilon = \{x : x \text{ is an odd integer and } 3 \leq x \leq 21\}$, $A = \{5, 7, 9, 17, 19\}$ and $B = \{3, 7, 11, 13\}$,
(a) find the value of $n(A \cup B)$,
(b) if the sets are represented on a Venn diagram, in which of the regions a, b, c and d will you place the element 11?



20. Given that $\varepsilon = \{x : x \text{ is a whole number and } x \leq 20\}$, $A = \{2, 4, 6, 8, 10, 12\}$ and $B = \{1, 4, 9, 16\}$, list the elements of each of the following.

(a) $A \cap B'$ **(b)** $A' \cap B$ **(c)** $A' \cap B'$ **(d)** $A' \cup B'$

21. Given that $\varepsilon = \{x : x \text{ is a positive integer and } 5 < 3x \leq 28\}$, $A = \{x : x \text{ is a multiple of } 3\}$ and $B = \{x : x \text{ is divisible by } 2\}$,

(a) find $\text{n}(A')$,

(b) list the elements of

(i) $(A \cup B)'$, **(ii)** $(A \cap B')$.

22. Given that $\varepsilon = \{(x, y) : x \text{ and } y \text{ are integers}\}$, $P = \{(x, y) : 0 < x \leq 3 \text{ and } 0 \leq y < 6\}$ and $Q = \{(x, y) : 2 \leq x < 8 \text{ and } 5 \leq y \leq 9\}$. List the members of the set $P \cap Q$.

23. If $\varepsilon = \{a, b, c, d, e, f, g\}$, $A = \{a, c, f, g\}$, $B = \{a, c, g\}$ and $C = \{b, c, e, f\}$,

(a) list the elements of

(i) $(A \cap B)'$, **(ii)** $A \cup C'$,

(b) find

(i) $\text{n}(A' \cap B')$, **(ii)** $\text{n}(B \cap C')$.

24. Given that $\varepsilon = \{1, 2, 3, 4, 5, ..., 19\}$, $A = \{x : x \text{ is prime}\}$, $B = \{x : x \text{ is a multiple of } 3\}$ and $C = \{x : x \text{ is a factor of } 12\}$,

(a) list the elements of the set

(i) A, **(ii)** C,

(b) find the value of $\text{n}(A \cup B)$.

25. The universal set ε is the set of all triangles. $A = \{\text{isosceles triangles}\}$, $B = \{\text{equilateral triangles}\}$, $C = \{\text{right-angled triangles}\}$ and $\varnothing$ is the empty set. Simplify the following.

(a) $A \cup B$ **(b)** $B \cap C$ **(c)** $A \cap B$

26. **(a)** If $A \subseteq B$ and $B \subseteq A$, what can you say about set A and set B?

(b) The three sets P, Q and R are such that $P \cap Q \neq \varnothing$, $P \cap R = \varnothing$ and $R \subseteq Q$. Draw a clearly labelled Venn diagram to illustrate the above information.

27. Given that $\varepsilon = \{x : x \text{ is an integer}\}$, $A = \{x : 20 < x \leq 32\}$ and $B = \{x : 24 \leq x \leq 37\}$, list the elements of

(a) $A \cap B$, **(b)** $A \cup B$.

28. Given that $\varepsilon = \{x : x \text{ is an integer and } 4 \leq x \leq 22\}$, $A = \{x : x \text{ is a multiple of } 5\}$, $B = \{x : x \text{ is a prime number}\}$ and $C = \{x : x \text{ is a factor of } 30\}$, list the elements of

(a) $A \cup C$, **(b)** $B \cap C$.

29. Given that $\varepsilon = \{6, 8, 10, 12, 13, 14, 15, 16, 18, 20, 21\}$, $A = \{x : x \text{ is a multiple of } 3\}$ and $B = \{x : 2x < 33\}$, find

(a) $A \cup B$, **(b)** $\text{n}(A \cap B')$.

30. Given that $\varepsilon = \{x : x \text{ is a natural number, } 2 \leq x \leq 15\}$, $A = \{x : x \text{ is a multiple of } 3\}$ and $B = \{x : x \text{ is even}\}$,

(a) list the elements of $A' \cap B$, **(b)** find the value of $\text{n}(A) - \text{n}(A \cap B)$.

31. Given that $\varepsilon = \{x : x \text{ is a positive integer}\}$, $A = \{x : 7 < 3x < 28\}$, $B = \{x : 3 < 2x + 1 < 25\}$ and $C = \{x : 1 < \frac{x}{2} \leq 9\}$,

(a) list the elements of the sets A, B and C,

(b) find the value of **(i)** $n(A \cup B)$, **(ii)** $n(B \cup C)$.

32. A, B and C are sets such that $A \cap B = \varnothing$ and $(A \cup B)' = C$. Simplify

(a) $A' \cap B$, **(b)** $A \cup B'$.

33. Given that $\varepsilon = \{x : x \text{ is a positive integer and } 20 \leq x \leq 90\}$, $A = \{x : x \text{ is a multiple of } 3\}$, $B = \{x : x \text{ is a perfect square}\}$ and $C = \{x : \text{unit digit of } x \text{ is } 1\}$,

(a) list the elements of

(i) $A \cap B$, **(ii)** $A \cap C$,

(b) find $n(B \cap C)$.

34. If $\varepsilon = \{x : x \text{ is an integer and } 0 \leq x \leq 24\}$, $A = \{x : x \text{ is a prime number}\}$ and $B = \{x : 12 < 3x < 37\}$, find

(a) $A \cap B$, **(b)** the value of $n(A \cap B)$.

35. State the number of elements in each of the following sets.

(a) $A = \{\text{factors of } 12\}$

(b) $B = \{\text{prime factors of } 48\}$

(c) $C = \{x : x \text{ is an integer and } 3x - 4 = 5\}$

(d) $D = \{x : x \text{ is a positive integer and } 3x - 7 < 33\}$

(e) $E = \{\text{a quadrilateral with 5 acute angles}\}$

(f) $F = \{x : x \text{ is a positive integer and } x^2 < 50\}$

36. Two sets P and Q are such that $P \cup Q = \varepsilon$ and $P \cap Q = Q$. Simplify the following.

(a) $P \cup Q$ **(b)** $Q \cap P'$

37. Given that A is a proper subset of B, simplify

(a) $A \cap B$, **(b)** $A \cup B$.

38. Given that $\varepsilon = \{\text{integers}\}$, $A = \{\text{factors of } 4\}$, $B = \{\text{factors of } 6\}$, $C = \{\text{factors of } 12\}$ and $D = \{\text{factors of } 9\}$, list the elements of each of the following.

(a) $A \cup B$ **(b)** $B \cap C$ **(c)** $C \cap D$

39. Given that $\varepsilon = \{\text{polygons}\}$, $A = \{\text{polygons with all sides equal}\}$, $B = \{\text{polygons with all angles equal}\}$, $C = \{\text{triangles}\}$ and $D = \{\text{quadrilaterals}\}$, state a name given to the members of each of the following.

(a) $A \cap C$ **(b)** $A \cap D$ **(c)** $B \cap D$

40. A, B and C are three non-empty sets satisfying the following conditions.

$$A \subset B,\ A \cap C \neq \varnothing \text{ and } C \not\subset B$$

Draw a clearly labelled Venn diagram to illustrate the above three sets.

41. Given that $\varepsilon = \{x : x \text{ is an integer less than } 22\}$, $A = \{x : x \text{ is a prime number less than } 20\}$ and $B = \{x : a < x < b\}$, find two pairs of values of a and b so that $A \cap B = \varnothing$.

42. Given that $A = \{(x, y) : x + y = 4\}$, $B = \{(x, y) : x = 2\}$ and $C = \{(x, y) : y = 2x\}$, list the elements of

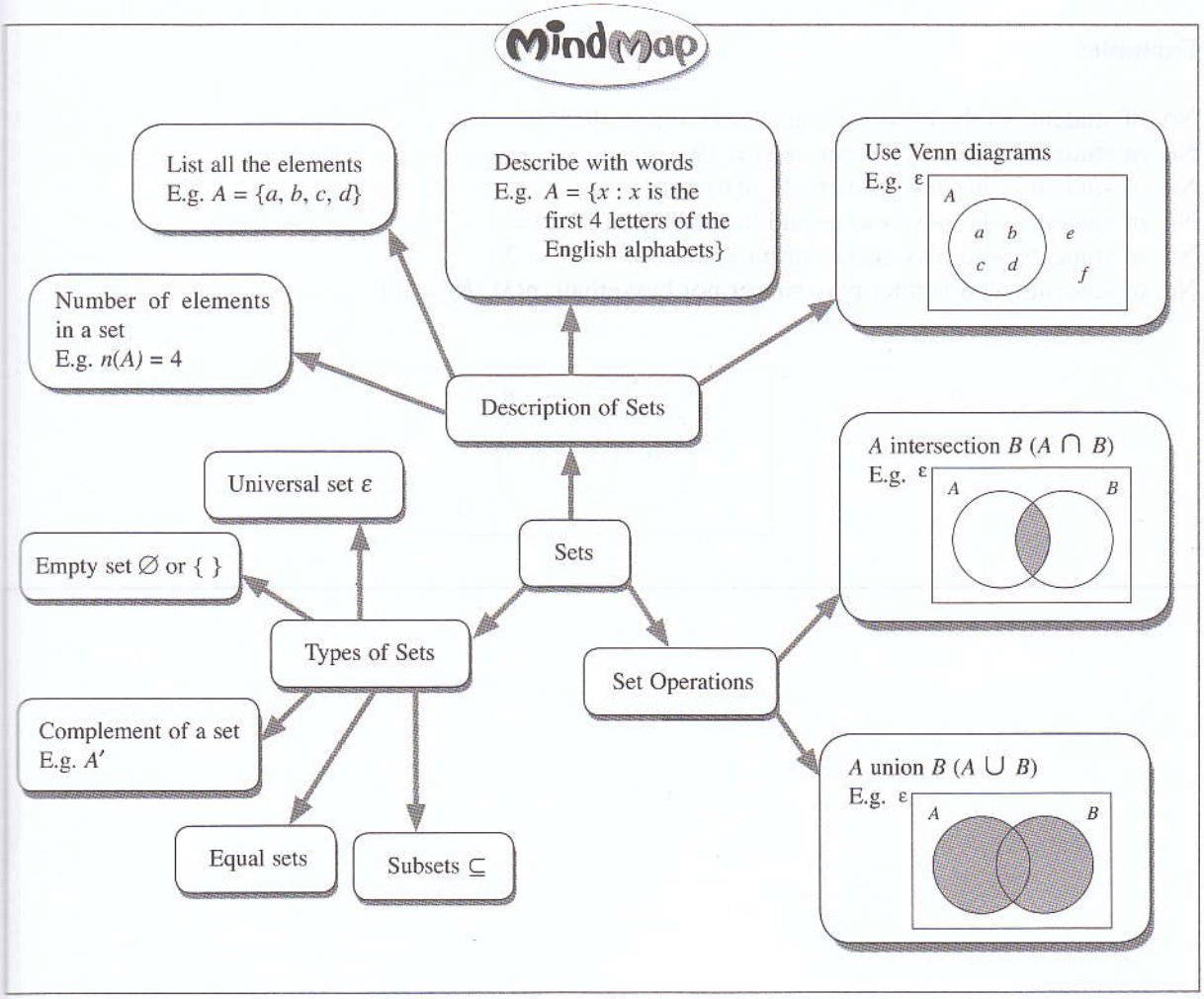
(a) $A \cap B$, **(b)** $B \cap C$, **(c)** $A \cap C$.

State the value of $n(A)$.

43. Draw separate Venn diagrams to illustrate each of the following relations between the sets A and B.

(a) $A' \cup B' = B'$ **(b)** $A \cap B = B$ **(c)** $A \cap B = \varnothing$ **(d)** $A' \cap B = \varnothing$

44. Draw a clearly labelled Venn diagram to represent the following information.
$\varepsilon = (a, b, c, d, e, f, g, h, i, j\}$, $A = \{a, c, e, f\}$, $B = \{b, c, d, e, h\}$, $C = \{d, e, f, i\}$.



Alternative ASSESSMENT

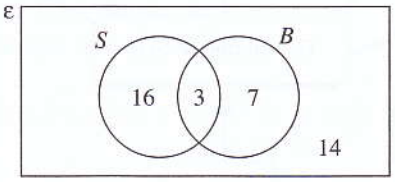
Performance Task: Classification of Real-life Objects

In real life, we can classify similar objects in sets and present them in the form of Venn diagrams for ease of visualisation. The Venn diagram below shows the number of students who play soccer and the number of students who play basketball in a class of 40 students.

Draw as many Venn diagrams as possible to represent objects in real life situations in the space provided below. Label your Venn Diagrams clearly and write down all the relevant set notations in words (see example below). Note that you should have a variety of different Venn Diagrams for different situations.

Example:

No. of students in the class (universal set), $n(\varepsilon) = 40$
No. of students who play soccer, $n(S) = 19$
No. of students who play basketball, $n(B) = 10$
No. of students who play soccer and basketball, $n(S \cap B) = 3$
No. of students who play soccer or basketball, $n(S \cup B) = 26$
No. of students who neither play soccer nor basketball, $n(S \cup B)' = 14$



Scoring Rubric:

Competency Level	Mathematical Concept	Creativity	Effort
4	Showed complete understanding of Venn diagrams and set notations	Drew a great variety of original and interesting Venn diagrams with detailed set notations	Put in a great deal of effort to draw many different types of Venn diagrams
3	Showed nearly complete understanding of Venn diagrams and set notations	Drew a variety of Venn diagrams with detailed set notations, some of which were rather common	Put in very good effort to draw different types of Venn diagrams
2	Showed some understanding of Venn diagrams and set notations	Drew some variety of Venn diagrams with set notations	Put in some good effort to draw some Venn diagrams
1	Showed limited understanding of Venn diagrams and set notations	Drew a limited variety of Venn diagrams with some set notations	Put in some effort in drawing a few Venn diagrams
0	Showed no understanding of Venn diagrams and set notations	Did not draw any Venn diagrams	Put in little or no effort; slipshod and tardy work
Score			

Final Score:

____ / 12

Final Score	10–12	8–9	6–7	4–5	0–3
Grade	A	B	C	D	F

Teacher's Comments (if any):

Chapter 11 Statistics

Summary

1. In a **dot diagram**, values are presented by **dots above a horizontal number line**.
2. In a **stem and leaf diagram**, a **value** is **split into** two parts, namely **a stem and a leaf**.
3. A set of data can be described by numerical quantities called **averages**.
4. The three common averages are the **mean**, the **median** and the **mode**.
5. The **mode** is the number that **occurs most frequently**.
6. The **mean** is the **sum of values divided by** the **number of values** in a set of data.
7. The **median** for an **odd** number of data **is the middle value** when the data are arranged in ascending/descending order. The **median** for an **even** number of data **is the mean of the two middle values** when the data are arranged in ascending/descending order.
8. The mean of a set of grouped data is $\bar{x} = \frac{\Sigma fx}{\Sigma f}$, where x is the mid-value of the class interval, and f is the frequency of the class interval.

Practice Questions

1. The marks out of ten scored by a class in a test are as given in the following table.

Marks	6	7	8	9	10
Number of students	2	5	10	12	6

(a) How many pupils sat for the test?
(b) Calculate the mean mark, correct to three significant figures.
(c) Find the median mark.
(d) Find the percentage of students who scored more than the median mark.

2. A record was kept of the number of packets of potato chips sold each day in a store and the results are as follows:

Number of packets	32	57	82	107	132	157	182
Number of days	3	5	8	7	10	6	1

(a) On how many days was a record kept of the number of packets of chips sold?
(b) Calculate the mean number of packets sold.
(c) Find the difference between the mode and the median.

3. The table below shows the number of errors made by Peter in typing a report.

Number of errors	0	1	2	3	4	5	6
Number of pages	1	3	10	7	4	3	2

(a) How many pages are there in Peter's report?
(b) What was the percentage of pages with less than 3 errors?
(c) What was the mode of the distribution?
(d) Calculate the mean number of errors made by Peter.

4. In a Mathematics test, the mean score of 30 students was 12.4. Mary, one of the 30 students, scored 8 marks. It later transpired that her score was recorded wrongly. After correcting Mary's score, the new mean score of the 30 students became 12.6. What was Mary's actual score?

5. The following are the heights (in metres) of a group of basketball players:

1.8, 1.9, 2.0, 1.7, 1.8, 1.9, 1.6, 2.0, 1.8, 1.9, 1.8.

(a) Find
(i) the modal height of the group,
(ii) the median height of the group,
(iii) the mean height of the group, correct to one decimal place.
(b) When the 12th member joined the group, the mean height became 1.9 m exactly. What was the height of the 12th member?

For Questions 6–9, find the mean, median and mode of the set of numbers.

6. 12, 11, 13, 11, 15, 16
7. 12, 18, 24, 20, 18, 11, 20, 29, 41, 20
8. 14.3, 13.5, 10.5, 12.6, 15.3, 16.4, 12.6, 16.0
9. 10.5, 9.6, 7, 11, 9.4, 8.1, 10.4, 11.7, 8.1, 9.4, 8.1

10. The mass of a group of children are 9, 11, 13, 13, 15, 15, 15, a, 18, 20.
Given that the median mass is 0.4 greater than the mean mass, find the value of a and state the modal mass.

11. A factory manufactures strapping machines. Over a fifteen-day period, the number of machines produced each day were 35, 38, 40, 45, 47, 45, 39, 45, 39, 38, 36, 43, 45, 42, 38.
Calculate
(a) the mean, **(b)** the modal, and
(c) the median number of strapping machines produced per day over this period.

12. The numbers 3, 7, 13, 14, 16, 19, 20 and x are arranged in ascending order. If the mean of the numbers is equal to the median, find x.

13. 18 swimmers were timed over a 100-metre distance. Their times, in seconds, were

63.1, 65.2, 65.1, 62.0, 67.1, 65.2, 64.3, 68.2, 65.9,
62.6, 64.2, 64.7, 62.0, 66.8, 65.2, 63.7, 67.4, 65.5.

Calculate
(a) the mean, **(b)** the modal and
(c) the median time of these swimmers.

14. The mean, the median and the mode of 4 numbers are 54, 56 and 60 respectively. Find the mean of the largest and the smallest numbers.

15. The table shows the number of fillings a class of 40 pupils had at the time of a dental inspection.

Number of fillings	0	1	2	3	4	5	6
Number of pupils	1	4	8	x	9	y	2

(a) If the mean number of the fillings per pupil is 3.2, find the values of x and y.
(b) If the mode is 4, find the largest possible value of x and calculate the mean number of fillings per pupil with x taking the largest value.

16. A gardener sowed 5 seeds into each of 100 plant pots. The number of seeds germinating in each pot was recorded and the results are as given in the table below.

Number of seeds germinating	0	1	2	3	4	5
Number of pots	10	30	25	20	10	5

(a) How many seeds did the gardener sow altogether?
(b) What fraction of the seeds germinated?
(c) Calculate the mean, median and mode of the distribution.

17. Thirty pupils were asked how many foreign countries they had visited. The answers are given below.

1	0	0	2	3	1	1	0	2	4
5	1	2	0	2	0	1	3	2	4
3	0	1	1	2	3	2	1	0	1

(a) Tabulate a frequency table for the above results.
(b) Find the modal, the median and the mean number of countries visited.

18. **(a)** The median of a set of eight numbers is 4.5. Given that seven of the numbers are 7, 2, 13, 4, 8, 2 and 1, find the eighth number and write down the mode of the eight numbers.
(b) The mean of a set of twelve numbers is 5 and the mean of a different set of eight numbers is a. Given that the mean of the combined set of twenty numbers is 8, calculate a.

19. For a certain question on a history examination paper, a candidate could score 0, 1, 2, 3 or 4 marks. The marks scored for this question by 30 students are shown in the table below.

Marks	0	1	2	3	4
Number of students	2	6	8	10	4

(a) Write down the modal mark. **(b)** Write down the median mark.
(c) Calculate the mean mark, given your answer correct to one decimal place.

20. A six-sided die is thrown 49 times. The results are shown in the table below.

Number shown on the die	1	2	3	4	5	6
Number of times	12	9	11	6	4	7

(a) For these results, write down
(i) the mode, **(ii)** the median.
(b) The die is thrown one more time. Find the number shown on the die if the mean of the 50 throws is to be exactly 3.

21. Copy and complete the table shown which gives the frequency distribution of the lengths of 40 fishes of a certain species, measured to the nearest mm.
Calculate the mean length of the fishes.

Length (mm)	*Mid-value*	*Frequency*
25–29	27	2
30–34		4
35–39		7
40–44		10
45–49		8
50–54		6
55–59		3

22. The table shown gives the frequency distribution of the marks obtained by 40 students in an English test.

(a) Write down the mid-value of the class interval $40 < x \leq 50$.

(b) Calculate the mean mark of the 40 students.

Marks (x)	*Number of students*
$20 < x \leq 30$	2
$30 < x \leq 40$	3
$40 < x \leq 50$	8
$50 < x \leq 60$	9
$60 < x \leq 70$	11
$70 < x \leq 80$	5
$80 < x \leq 90$	2

23. The mass, in kg, of 80 members of a sports club were measured and recorded as shown in the table.

Calculate the mean mass of the 80 members.

Mass (x kg)	*Number of members*
$40 < x \leq 50$	7
$50 < x \leq 60$	10
$60 < x \leq 70$	14
$70 < x \leq 80$	27
$80 < x \leq 90$	12
$90 < x \leq 100$	6
$100 < x \leq 110$	4

24. The table gives the frequency distribution of the mass of 200 steel bars, to the nearest kg.

Calculate the mean mass of the 200 steel bars.

Mass (kg)	*Number of steel bars*
10–29	32
30–39	38
40–49	64
50–59	35
60–69	22
70–99	9

25. The following diagram represents the scores of 20 students in an examination for two subjects.

English						Chinese					
Leaf					**Stem**	**Leaf**					
			5	3	3	9					
	8	6	6	6	4	0	6				
		7	6	3	5	2	3	7	9		
8	7	6	4	0	6	1	3	4	8		
8	7	5	4	2	7	0	2	2	5	7	9
				1	8	0	0	2	9		

(a) Calculate the mean score for each subject.
(b) State the median score for each subject.
(c) Comment briefly if John scored 64 in both subjects.

26. The police force records the number of emergency calls per day in 31 days in October and December.

October								December										
Leaf							**Stem**	**Leaf**										
	8	7	7	5	4	4	4	0	8									
			7	1	1	0	0	1	0	3	4							
9	9	8	7	6	6	5	5	5	2	0	0	1	1	1	1	8		
				6	5	3	3	3	2	3	3	5	7	7	7	9		
		5	4	1	0	0	0	4	6	6	7	8	8	8	8	9	9	9	9	9

(a) State the modal number of emergency calls in December.
(b) State the median number of calls in each month.
(c) Calculate the mean number of emergency calls in each month.
(d) Comment briefly on what the data indicates for the two months.

27. A sample of 20 light bulbs from two different brands is kept switched on and the number of hours they last is recorded.

Brand *X*: 442 447 550 554 557 558 660 664 665 665
771 772 773 773 777 779 880 883 883 888

Brand *Y*: 433 437 445 446 446 449 553 557 557 661
665 665 669 669 772 773 775 776 776 882

(a) Using the data above, copy and complete the stem and leaf diagram given.

Brand X Leaf	Stem	Brand Y Leaf
47 42	4	33 37
50	5	53
	6	
	7	
	8	

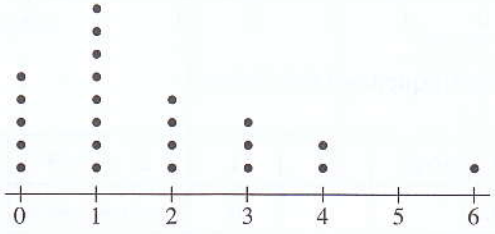
(b) Which brand of light bulbs last for the "greatest number of hours"?
(c) Which brand of light bulbs last for the "least number of hours"?
(d) Comment on the distribution of data of each brand.

28. The mass of a group of university students were recorded. Below is a stem-and-leaf diagram of the mass (in kg) of the students.

Stem	Leaf
40	4 7 3
50	0 5 6 7 4 9 2 4 1
60	3 3 1 7 9 8 5 2 3 6 3
70	4 2 0 8 9 6 7 1 0 4 4 8 9
80	9 0 2 4 3 7 8 6 7
90	8 7 6 4 0 6
100	3 0 2 8 9

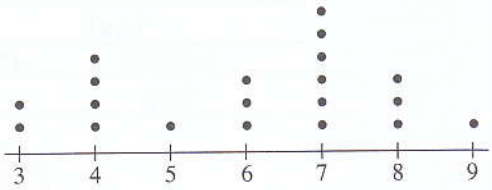
(a) How many university students were there?
(b) If the heaviest student was 100.9 kg, write down the mass of the lightest student.
(c) What is the most common mass of the students?
(d) The university encourages students to do more exercise to reduce their mass below 70.5 kg, otherwise, they are considered as overweight. Find the percentage of the number of students who are considered overweight.

29. The dot diagram below shows the number of siblings a child has as found in a survey.



(a) How many children had participated in the survey?
(b) What is the largest number of children in a family?
(c) What is the average number of children in a family in the survey?

30. The dot diagram represents the lengths, in cm, of 20 leaves.

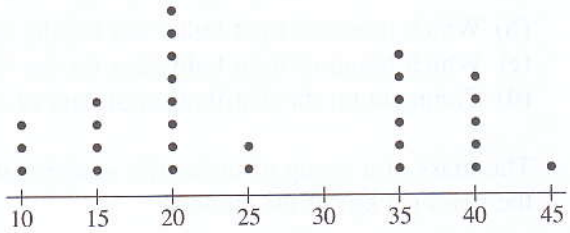


(a) What is the most common length of the leaves?

(b) What is the longest length of the leaves?

(c) What is the percentage of leaves whose length is more than 6 cm?

31. The dot diagram represents the travelling time in minutes, from home to school, of some students.



(a) How many students are there?

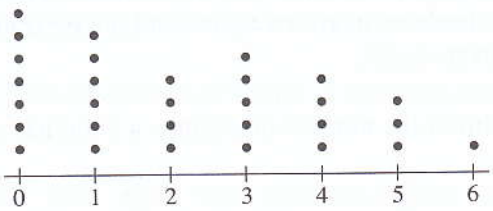
(b) What is the most common travelling time?

(c) What is the percentage of students who take less than half an hour to reach the school? Comment briefly on what the data indicates.

(d) Complete the frequency table in the answer space.

Travelling time (minutes)	10	15	20	25	30	35	40	45
Number of students	3	4						1

32. The following diagram represents the number of goals scored by a football team in each of the 30 matches.



(a) Copy and complete the frequency table below.

Number of goals scored	0	1	2	3	4	5	6
Number of matches		6		5			

(b) What is the name of the above diagram?

(c) Find the mode, median and mean.

(d) Comment briefly the performance of this football team.

MindMap

Mean = $\frac{\text{Sum of the values}}{\text{Total number of values}}$

e.g. Given that the scores of 5 students are 95, 67, 82, 74, 51

Mean = $\frac{95 + 67 + 82 + 74 + 51}{5} = 73.8$

Measure of Central Tendency

Median: First arrange the numbers in ascending order, then

(a) the median for an **odd** number of data is the middle value.

e.g. 3, 4, 4, 5, 6, 9, 9

median = 5

(b) the median for an **even** number of data is the **average** of the two middle values.

e.g. 3, 4, 4, 5, 6, 9

median = $\frac{4 + 5}{2} = 4.5$

Mode:

The number which occurs most frequently in a set of numbers.

e.g. (1) 0, 1, 1, 2, 4, 4, 4, 9

mode = 4

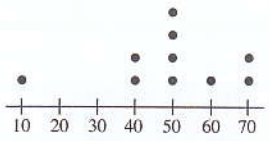
e.g. (2) 0, 1, 1, 1, 2, 4, 4, 4, 9, 9

mode = 1 and 4

Statistical Diagram

Given data: 10, 40, 40, 50, 50, 50, 50, 60, 70, 70

Dot Diagram



Given data: 36, 40, 40, 40, 41, 41, 50, 53, 58, 59, 62, 62, 65

Stem and Leaf Diagram

Stem	Leaf
6	2 2 5
5	0 3 8 9
4	0 0 0 1 1
3	6

Alternative ASSESSMENT

Journal Writing: Choosing the Right Pictorial Representation

Statistics is the science of *collecting*, *organising*, *displaying* and *interpreting* numerical data. There are many types of graphical representation that we can use to display the data but we have to decide on the form of graphical representation which is the most suitable for the type of data we have.

What are some of the advantages and disadvantages of displaying the data in the following ways? Complete the table below. [1 mark for each box]

Graphical Representation	Advantages	Disadvantages
Pictograms		
Dot Diagrams or Dot Plots		
Bar Graphs or Bar Charts		

Graphical Representation	Advantages	Disadvantages
Pie Charts		
Line Graphs		
Histograms		
Stem and Leaf Diagrams or Stem Plots		

Final Score:

____ / 14

Final Score	12–14	9–11	7–8	5–6	0–4
Grade	A	B	C	D	F

Teacher's Comments (if any):

Mathematical Investigation: When the Mean of Averages Can Be Used

The purpose of this worksheet is to investigate when you can or cannot take the mean of averages.

Section A: Mean of Average Scores

1. The average scores in the Mathematics final exam for three Sec 2 classes in a particular school are 80, 65 and 50, out of 100 marks. The number of pupils in the three classes are 30, 40 and 44 respectively. Find the average score in the Mathematics final exam for the three classes. **[2]**

2. What if you just take the mean of the three average scores 80, 65 and 50? Will this give the average score of the three classes? Why or why not? **[2]**

3. What lesson can you learn from this problem? Are there any exceptions? If yes, give an example. **[2]**

Section B: Mean of Percentages

4. Alice scored 75% and 60% for her Additional Maths Exam Paper 1 and Paper 2 respectively. But the two papers have unequal weightage. The full score for Paper 1 is 80 marks but that for Paper 2 is 100 marks. The total score for both papers is 80 + 100 = 180 marks. Alice believed that her mean score for both papers is $\frac{75\% + 60\%}{2} = 67.5\%$. But her teacher said that she was wrong. Can you calculate Alice's actual mean score for both papers and explain to her why she was wrong in her previous calculation? **[3]**

5. Beng Seng sold two paintings for $300 each, one at a profit of 20% on his cost and the other at a loss of 20% on his cost. Did he gain, lose or break even? Why? **[4]**

6. What lesson can you learn from these two problems in this section? Are there any exceptions? If yes, give an example. **[2]**

Section C: Mean of Speeds

7. Ali travelled at 100 km/h for the first half of a 10-km journey. Then he travelled at 50 km/h for the rest of his journey. What was his average speed for the whole journey? **[2]**

8. What if you just take the mean of the two speeds 100 km/h and 50 km/h since Ali travelled the same distance of 5 km for both parts of the journey? Will this give the average speed for the whole journey? Why or why not? **[2]**

9. What lesson can you learn from this section? Are there any exceptions? If yes, give an example. [2]

Section D: Conclusion

10. Summarise the three main lessons that you have learnt from this worksheet. [3]

Final Score: ____ / 24

Final Score	20–24	16–19	12–15	8–11	0–7
Grade	A	B	C	D	F

Teacher's Comments (if any):

Mathematical Investigation: Does Statistics Lie?

Statistics is the science of *collecting*, *organising*, *displaying* and *interpreting* numerical data. Evan Esar (1899–1995) once commented, "Statistics is the only science that enables different experts using the same figures to draw different conclusions." Benjamin Disraeli (1804–1881) also remarked, "There are 3 kinds of lies: lies, damned lies and statistics." So does statistics lie? This is the main purpose of this worksheet: to investigate whether statistics can lie.

Section A: Collection of Data

1. Alice and Edwin are each asked by their teacher to survey 200 Singaporeans on whether they like shopping. Their findings are as shown below.

	Alice's Data	Edwin's Data
Number of people who like shopping	128	29
Number of people who are neutral	47	24
Number of people who dislike shopping	25	147

(a) According to Alice's data, do most Singaporeans like shopping? _____

(b) According to Edwin's data, do most Singaporeans like shopping? _____ **[1]**

2. Think of as many reasons as possible why there is such a big discrepancy in their data. **[2]**

3. What is the **main** problem with Alice's and Edwin's surveys? **[1]**

Section B: Organisation of Data

4. In a survey reported in a newspaper, the first two paragraphs read:

> **Insurance Firms, Banks, Top Hate List**
>
> Banks and insurance companies have made it to the top of the consumer hate list for the first time. They were the target of 1915 complaints to the Consumers Association of Singapore (Case) between January and last month, edging out the usual suspects – timeshare companies (1228 complaints), motor vehicle shops and companies (1027), renovation companies (963), and electrical and electronics shops (710).

Do you agree that banks and insurance companies have received more complaints than timeshare companies? Why or why not? **[2]**

_____

_____

_____

_____

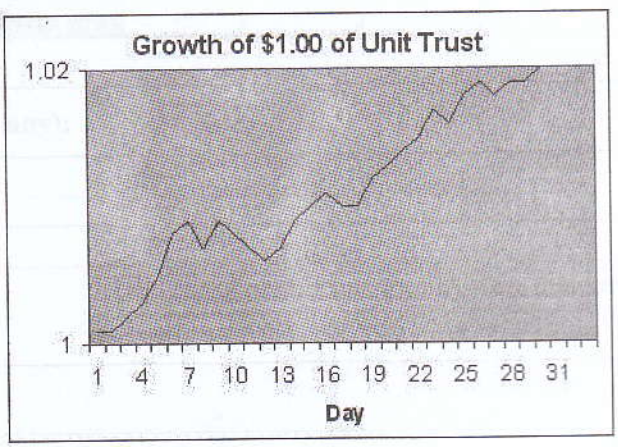
5. What is the **main** problem with this survey report? **[1]**

_____

_____

Section C: Display of Data

6. The following line graph was used by a salesman to promote the sales of unit trust.



According to the salesman, the line graph shows that there is a large increase in the value of the unit trust from Day 1 to Day 31 of the particular month. So if you were to buy this unit trust, it is very profitable. Do you agree? Why or why not? **[2]**

7. What is the **main** problem with the salesman's line graph? **[1]**

Section D: Interpretation of Data

8. In a survey reported in a newspaper, the last part of the report read:

> **Survey finds discipline in schools not a big problem**
>
> As it turned out, three out of five teachers in the survey said the discipline problem was not serious. "This is the good news – that the state of discipline in schools is not as bad as it has been made out to be." The survey interviewed 285 teachers over two years.

Do you agree with the conclusion? Why or why not? **[2]**

9. What is the **main** problem with the conclusion of this survey? **[1]**

Section E: Conclusion

10. Mark Twain (1835–1910) said, "Get your facts first, and then you can distort them as much as you please. Facts are stubborn, but statistics are more pliable." Do you agree? Why or why not? **[2]**

Final Score:

_____ / 15

Final Score	12–15	10–11	8–9	6–7	0–5
Grade	A	B	C	D	F

Teacher's Comments (if any):

Chapter 12 Probability

Summary

1. A **sample space** or **probability space** is the collection of all possible outcomes of a probability experiment.

2. An event E contains the outcomes from the sample space that favour the occurrence of the event.

3. In a probability experiment with m equally likely outcomes, if k of these outcomes favour occurrence of an event E, then the **probability** of the event E happening is:

$$P(E) = \frac{\text{Number of favourable outcomes for event } E}{\text{Total number of possible outcomes}} = \frac{n(E)}{n(S)} = \frac{k}{m}$$

where $n(E)$ is the number of favourable outcomes in the event E and $n(S)$ is the total number of possible outcomes.

4. For any event E, $0 \leq P(E) \leq 1$.
$P(E) = 0$ if and only if the event E cannot possibly occur.
$P(E) = 1$ if and only if the event E will certainly occur.

Practice Questions

1. A card is drawn at random from a pack of 52 ordinary playing cards. Find the probability of drawing
   (a) a King, **(b)** the King of diamonds,
   (c) a heart, **(d)** a picture card.

2. A fair die is tossed once. Find the probability of obtaining
   (a) an odd number, **(b)** a number less than four,
   (c) a five or six, **(d)** a number which is not a six.

3. A bag contains 3 red balls and 5 yellow balls.
   (a) Find the probability of selecting at random
   (i) a red ball, **(ii)** a yellow ball.
   (b) One yellow ball is removed from the bag. Find the new probability of selecting at random
   (i) a red ball, **(ii)** a yellow ball.

4. A class has 12 boys and 28 girls.

(a) Find the probability of choosing at random

(i) a boy, **(ii)** a girl.

(b) One girl leaves the class. Find the new probability of selecting at random

(i) a boy, **(ii)** a girl.

5. A 20¢ coin and a 50¢ coin are tossed at the same time. If S is the sample space, list all the possible outcomes. Find the probability of obtaining

(a) two tails, **(b)** a head and a tail.

6. Each letter of the word "*INDEPENDENT*" is written on individual cards. The cards are placed in a box and mixed thoroughly. A card is then picked at random from the box.

(a) Find the probability of picking a card with

(i) the letter P, **(ii)** the letter E,

(iii) a vowel, **(iv)** a consonant.

(b) One card with the letter E is removed from the box. Find the new probability of picking a card with

(i) the letter P, **(ii)** the letter E,

(iii) a vowel, **(iv)** a consonant.

7. A solid in the shape of a regular tetrahedron (four sides) has the colours red, blue, yellow and green on its faces. The numbers 2, 3, 4 and 5 are labelled on the red, blue, yellow and green faces respectively. The solid is tossed once. Find the probability that it lands on

(a) the red face,

(b) the blue or yellow face,

(c) the face labelled with a prime number.

8. A roulette wheel has slots numbered from 0 to 36. Assuming that the wheel is fair, find the probability that the ball lands in the slot numbered

(a) 13, **(b)** with a prime number,

(c) with a number less than 19, **(d)** with a number which is a multiple of 4,

(e) with an odd number.

9. A bag of sweets contains 7 toffees, 4 barley sugars and 10 chocolates.

(a) Find the probability of selecting at random

(i) a toffee,

(ii) a toffee or a chocolate,

(iii) a barley sugar or a chocolate.

(b) One toffee is removed from the box. Find the new probability of selecting

(i) a toffee,

(ii) a toffee or a chocolate,

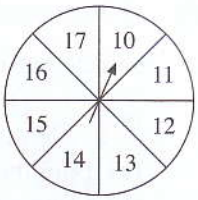
(iii) a barley sugar or a chocolate.

10. A poker die has 6 faces representing the cards of an ordinary pack: 9, 10, J, Q, K and Ace, each of the same suit. The die is tossed once. Find the probability of obtaining a face representing

(a) a picture card,

(b) a card "higher than" J, with Ace being the highest.

11. The diagram shows a spinner. The pointer is spun once. Find the probability that it points at



(a) a prime number,
(b) an even number,
(c) a number less than 15.

12. The set $S = \{n : n \text{ is an integer such that } 1 \leq n \leq 50\}$.
(a) Find how many elements of S contain the digit "2" at least once.
(b) If an element of S is selected at random, find the probability that it
(i) is a prime number, **(ii)** is an odd number,
(iii) is a multiple of 5, **(iv)** does not contain the digit "2" at all,
(v) is a perfect square, **(vi)** is not divisible by either 2 or 3.

13. In a test, the marks obtained by 15 pupils are 42, 44, 38, 39, 44, 45, 47, 48, 42, 36, 44, 40, 39, 34 and 48.
(a) Find the probability that a pupil chosen at random scored a mark which is
(i) not a prime number, **(ii)** divisible by 11,
(iii) less than 44, **(v)** divisible by 3.
(b) The pass mark of the test was 41.
(i) Find the probability that a pupil chosen at random passed the test.
(ii) A pupil was chosen at random from those who failed. Find the probability that the pupil's mark was 39.

14. A box contains 8 cards numbered 7, 15, 17, 21, 24, 25, 29 and 30. A card is selected at random from the box. Find the probability that the number on the card
(a) is divisible by 3,
(b) is a prime number,
(c) has a sum that is divisible by 2.

15. A bag contains 18 red and x white discs. The probability that a disc drawn at random from the bag will be white is $\frac{3}{5}$.
(a) Find the value of x.
(b) If 10 more red and 15 blue discs are added to the bag, find the probability that a disc selected at random will be blue.

16. A machine generates a two-digit number randomly. Find the probability that the number generated
(a) is greater than 87, **(b)** is less than 23,
(c) is divisible by 4, **(d)** is between 55 and 72 both inclusive.

17. A container contains 8000 1-kg bags of sugar. If the probability that a bag of sugar selected at random will weigh more than 1 kg is $\frac{1}{40}$ while the probability that a bag selected at random will weigh less than 1 kg is $\frac{1}{160}$, find the probability that a bag selected at random will weigh exactly 1 kg. How many bags of sugar are there, each of which weighs less than 1 kg?

18. A book shelf contains 46 Science books, 24 History books and x Mathematics books. A book is selected at random from the book shelf and the probability that a Mathematics book is selected is $\frac{3}{8}$.

(a) Find the value of x.

(b) Hence, find the probability that when a book is selected, it will be

(i) a History book, **(ii)** a Geography book, **(iii)** a Science book.

19. A bag contains x red balls, $(x + 5)$ blue balls and $(3x + 10)$ white balls. If the probability of drawing a blue ball is $\frac{2}{9}$,

(a) find the value of x.

(b) Hence, find the probability of drawing a

(i) red ball, **(ii)** green ball, **(iii)** white ball.

20. The table below shows the number of each type of school personnel in a Singapore Secondary School.

School Personnel	P, V.P, HODs	Teachers	Lab & Technical	Clerical
No. of School Personnel	16	64	8	12

If a school personnel is selected randomly from the school, find the probability that he/she is

(a) a laboratory or technical personnel, **(b)** a P, VP or HOD,

(c) a clerical officer.

21. In a carpark, there are 85 cars and 25 pickups. After x cars have left the carpark, the probability that a pickup leaves the carpark next is $\frac{5}{18}$. Calculate the value of x.

22. There are 25 red balls, 15 blue balls and x black balls in a bag. A ball is drawn at random from the bag. Given that the probability of drawing a blue ball is $\frac{5}{12}$, find the value of x and hence find the probability of drawing a black ball from the bag.

23. There are 36 white marbles and 12 red marbles in a bag. Write down the probability of drawing a red marble from the bag.

After $2x$ white marbles and $(x + 2)$ red marbles are added to the bag, the probability of selecting a red marble from the bag becomes $\frac{3}{10}$.

Calculate the value of x.

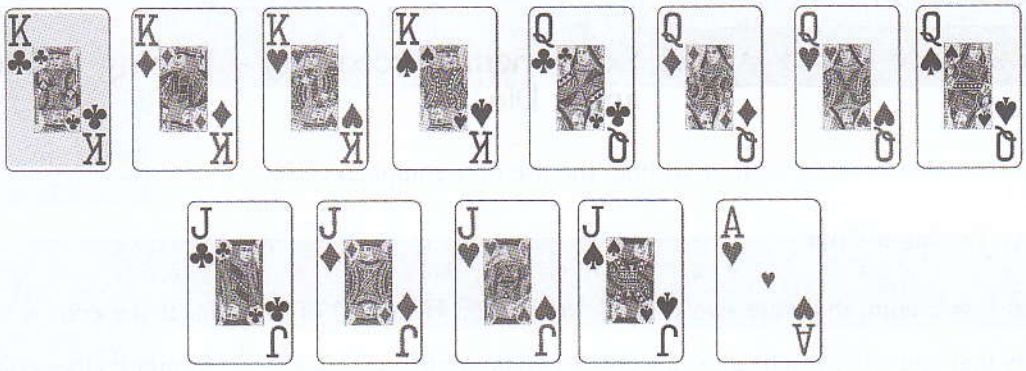
24. A box contains the following 7 cards: $\boxed{A}\ \boxed{B}\ \boxed{B}\ \boxed{C}\ \boxed{C}\ \boxed{C}\ \boxed{D}$. A card is selected at random.

(a) Find the probability that the card selected bears

(i) a vowel, **(ii)** the letter C.

(b) x cards each bearing the letter A are added to the box and the probability that a card drawn from the box is $\boxed{C}$ becomes $\frac{1}{7}$. Find the value of x.

25. One card is selected at random from the set of cards shown below.



Find the probability of selecting

(a) the Jack of diamonds,

(b) a King, Queen or Jack,

(c) the ace of hearts or King of hearts,

(d) a joker.

26. Each of the letters in the sentence "I LOVE MATHEMATICS" is written on a card and all the cards are put in a box. A card is selected at random from the box. Find the probability of selecting

(a) a vowel,

(b) a letter which appears in the word : "SCIENCE",

(c) a letter which appears in the word: "SMART",

(d) a letter which appears in the word "DUG".

27. The Venn diagram shows the number of pupils in a class of 35, where D = {pupils who can dance}, G = {pupils who can play the guitar}.
A pupil is selected at random from the class. Find the probability that the pupil

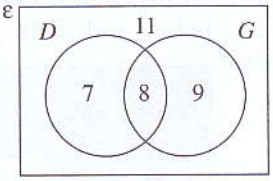
(a) can dance,

(b) can play the guitar,

(c) can dance as well as play the guitar,

(d) can neither dance nor play the guitar,

(e) can play the guitar but cannot dance.



28. The Venn diagram below shows the number of Sec 2 pupils in a secondary school, where

M = {pupils who like Mathematics},

S = {pupils who like Science}.

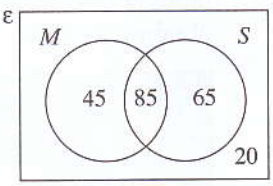
A pupil is selected at random from the group. Find the probability that the pupil likes

(a) both Science and Mathematics,

(b) Mathematics but not Science,

(c) neither Science nor Mathematics,

(d) Science.



Alternative ASSESSMENT

Exploratory or IT Worksheet: Experimental Probability – Tossing a Coin and a Die

This worksheet can be done with or without the use of a computer.

Section A: Tossing a Coin

When you toss a coin, there are two possible outcomes: Head (H) or Tail (T). If the coin is fair, the probability that you will get a head is one out of two possible outcomes or $\frac{1}{2}$. Similarly, the probability that you will get a tail is $\frac{1}{2}$.

1. Suppose you toss a coin and the outcome is a head. Then you toss the coin again. Will you definitely get a tail this time? Why or why not? [1]

2. Suppose you toss a coin 10 times. Will you definitely get 5 heads and 5 tails? [1]

3. Now take a coin and toss it 10 times. Record the number of heads and tails obtained in the table below. Calculate the probability of obtaining a head or a tail for your 10 tosses. [2]

Outcome	Tally	Number of heads or tails for 10 tosses	Probability of getting a head or a tail
Head			
Tail			

4. If your classmates are doing the same probability experiment as you, you can combine your results with one of them to get the number of heads or tails for 20 tosses and record them in the table below; or you can toss the coin for another 10 times yourself. Calculate the probability of obtaining a head or a tail for your 20 tosses. [2]

Outcome	Tally (if you are tossing yourself)	Number of heads or tails for 20 tosses	Probability of getting a head or a tail
Head			
Tail			

5. Repeat for 30, 40 and 50 tosses and record your results in the tables below. [3]

Outcome	Tally (if you are tossing yourself)	Number of heads or tails for 30 tosses	Probability of getting a head or a tail
Head			
Tail			

Outcome	Tally (if you are tossing yourself)	Number of heads or tails for 40 tosses	Probability of getting a head or a tail
Head			
Tail			

Outcome	Tally (if you are tossing yourself)	Number of heads or tails for 50 tosses	Probability of getting a head or a tail
Head			
Tail			

6. What do you notice about the last column in the five tables above? Do the probabilities of getting a head or a tail approach $\frac{1}{2}$ when there are more tosses? [1]

7. If you toss a coin 1000 times, will you expect to get *exactly* 500 heads and *exactly* 500 tails? If yes, explain why. If not, state what you will expect to get. [1]

Section B: Tossing a Die

When you toss a die, there are six possible outcomes: 1, 2, 3, 4, 5 or 6. If the die is fair, the probability that you will get a '3' is one out of six possible outcomes or $\frac{1}{6}$. Similarly, the probability that you will get a '4' is $\frac{1}{6}$.

Do you know that the singular for 'dice' is 'die'? That is why if you are addicted to gambling, you will surely die!

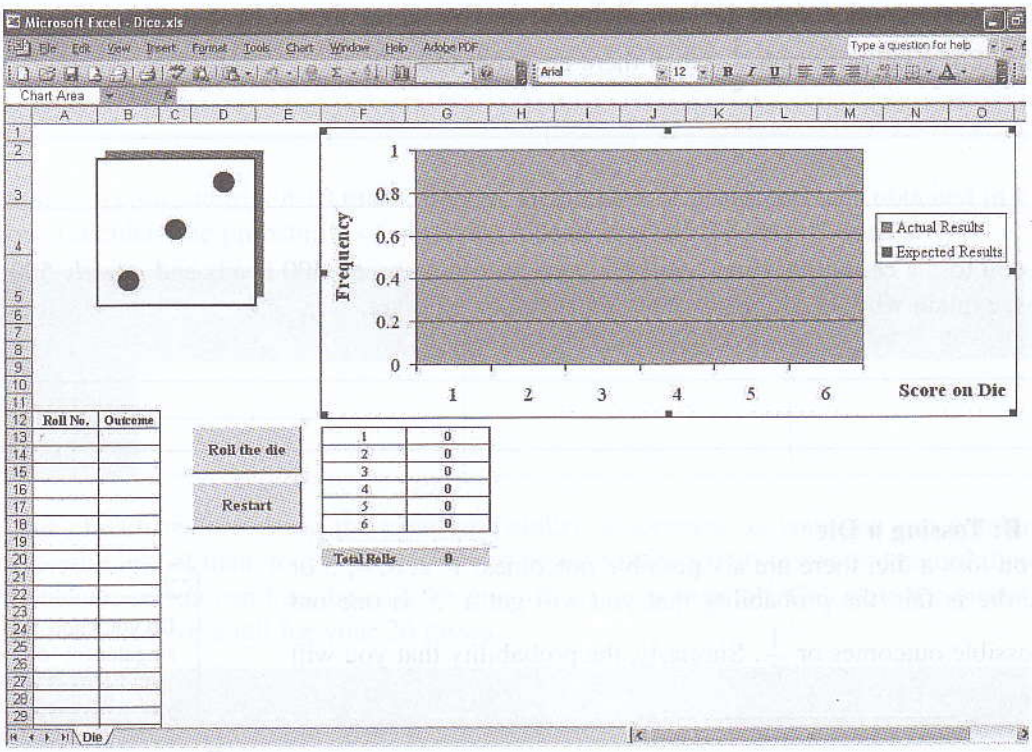
8. Suppose you toss a die 5 times and the outcomes are a '1',a '2', a '3', a '4' and a '5'. Then you toss the coin the sixth time. Will you definitely get a '6' this time? Why or why not? **[1]**

9. Suppose you toss a die 6 times. Will you definitely get at least a '2' in the six outcomes? **[1]**

Open the appropriate template from the Workbook CD and click ***Enable Macros*** when the dialogue box "Security Warning" appears. It needs macros to work.

If the dialogue box says that "Macros are disabled...", then click ***OK***, and go to the ***Tools*** menu and select ***Options***. Then select the ***Security*** tab and ***Macro Security***. Then select ***Security Level*** tab and choose ***Medium***. Click ***OK*** and close the Excel file. Reopen the template and select ***Enable Macros***.

If you don't have a computer, just use a normal die.



10. Click on the button 'Roll the Die' and it will roll the die once. Repeat for a total of 20 tosses and record the number of '1', '2', '3', '4', '5' and '6' obtained in the table below. Calculate the probability of obtaining each of the six outcomes for your 20 tosses. **[2]**

Outcome	Tally (if you are not using computer)	Number of corresponding outcome for 20 tosses	Probability of getting corresponding outcome for 20 tosses
'1'			
'2'			
'3'			
'4'			
'5'			
'6'			

11. Repeat to get 200 tosses if you are using the computer (or 50 tosses if you are not) and record your results in the table below. **[2]**

Outcome	Tally (if you are not using computer)	Number of corresponding outcome for 200 tosses	Probability of getting corresponding outcome for 200 tosses
'1'			
'2'			
'3'			
'4'			
'5'			
'6'			

12. What do you notice about the last column in the two tables above? Do the probabilities of getting any one of the six outcomes approach $\frac{1}{6}$ when there are more tosses? **[1]**

13. If you toss a die 600 times, will you expect to get *exactly* 100 'sixes'? If yes, explain why. If not, state what you will expect to get. **[1]**

Section C: Conclusion

14. Write down one main lesson that you have learnt from this worksheet. **[1]**

Final Score:

☐ **/ 20**

Final Score	16–20	13–15	10–12	7–9	0–6
Grade	A	B	C	D	F

Teacher's Comments (if any):

Term IV Revision Test — Time: $1\frac{1}{2}$ h

1. Given that ε = {integers from 1 to 29 inclusive}. P = {prime numbers}, $N = \{x : x > 9\}$, M = {multiples of 3}, K = {multiples of 5}, write down the members of the following sets.

(a) $P \cap N$ **[1]**

(b) $K \cap M$ **[1]**

(c) $M \cap P'$ **[2]**

2. Given that ε = {integers}, A = {factors of 8}, B = {factors of 12}, C = {multiples of 2}, D = {multiples of 3}, find

(a) $A \cup B$, **(b)** $B \cup C$,

(c) $n(B \cap D)$, **(d)** $n(A \cap C)$. **[4]**

3. A bag contains x red balls and $(2x + 3)$ blue balls. If the probability that a ball drawn at random will be blue is $\frac{5}{7}$, find the value of x. **[3]**

4. In a class of 16 boys and 24 girls, 6 of the boys and 12 of the girls are short-sighted. A pupil is chosen at random from the class. Find the probability that the pupil chosen is

(a) a boy,

(b) short-sighted,

(c) a short-sighted girl. **[3]**

5. Fifteen children were asked to guess the mass of a cake to the nearest $\frac{1}{2}$ kg and the results are as follows:

$4\frac{1}{2}$, 4, $3\frac{1}{2}$, $3\frac{1}{2}$, $2\frac{1}{2}$, 3, $4\frac{1}{2}$, 4, $4\frac{1}{2}$, $2\frac{1}{2}$, $4\frac{1}{2}$, 2, 4, $3\frac{1}{2}$, $4\frac{1}{2}$.

Find

(a) the modal,

(b) the median, and

(c) the mean

value of the results. **[4]**

6. **(a)** Given that $\frac{5q - pr}{3qr - 4p} = \frac{2}{3}$, express r in terms of p and q.

(b) Factorise completely $2x^4y - 18x^2y^3$.

(c) Solve the equation $4 - 4x + \frac{3}{x} = 0$. **[7]**

7. The mean height of 21 boys and 17 girls is 161 cm. If the mean height of the 17 girls is 152 cm, what is the mean height of the 21 boys? **[2]**

8. The time (in minutes) to complete a Mathematics worksheet by 36 pupils are as follows.

Stem	Leaf
1	6 7 8 8 9 9 9
2	1 3 3 4 5 5 7 8 8 9 9 9
3	2 2 3 3 4 5 6 6 8 8 9
4	0 1 1 2 5 8

Find the

(a) mean, **(b)** mode. **[2]**

9. Each of the 50 pupils in a school was asked how many coins he had in his pocket. The results are given in the table below.

Number of coins	0	1	2	3	4	5	6	7	8	9
Frequency	6	8	9	8	7	6	3	2	1	0

(a) Find

(i) the total number of coins these pupils had in their pockets,

(ii) the mean number of coins per pupil.

(b) What is the

(i) mode, and

(ii) median of the distribution? **[5]**

10. In 55 games of basketball, 'Fast Jason' scored the number of points recorded as shown.

12	19	14	13	10	11	14	13	11	15	12
13	16	14	15	16	10	12	13	15	12	13
16	17	12	11	13	12	12	11	15	14	11
13	15	17	14	13	10	12	11	13	13	16
16	12	12	13	15	16	13	10	15	11	10

(a) Organise this data into a frequency distribution table.

(b) Construct a histogram for Fast Jason's scores. Which of the mode, mean or median is indicated by the highest column in your histogram?

(c) Determine the mode and median of Fast Jason's scores. **[6]**

11. The time (in minutes) that a random sample of 40 pupils take to travel from home to school are as shown below.

Stem	Leaf
0	7 8
1	2 2 2 3 5 5 7 7 7 8
2	3 3 3 3 3 4 6 9
3	2 5 5 7 8 9
4	1 3 4 4 4 6 6
5	6 6 6 6
6	8
7	3 4

(a) What is the average number of minutes that the 40 pupils take to travel to school?

(b) What is the most common time taken by the pupils to travel to school? **[4]**

12. The school bags of a group of primary five pupils were weighed. The stem-and-leaf diagram shown represents the mass (in kg) of the school bags of the group of primary five pupils.

3	9	7	4	1	6	0					
4	3	4	7								
5	6	5	7	9	4	2	1	0	4		
6	8	7	9	3	1	3	6	5	2	3	3
7	4	0	6	8	2	9	4	7	1	8	0	4
8	8	6	3	2	7	4	0	9			
9	6	8	7	4							
10	3	0	8	2							

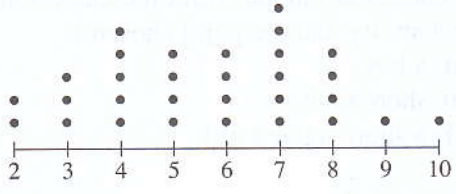
(a) How many primary five pupils were there?

(b) If the heaviest school bag was 10.8 kg, write down the mass of the lightest school bag.

(c) What was the most common mass of school bags carried by these pupils?

(d) The school encourages pupils to carry bags weighing less than 7.5 kg. Find the percentage of school bags that were considered 'overweight'. **[5]**

13. A group of pupils were asked to count the total number of pens and pencils in their pencil cases. The following are the results.



Number of pens and pencils

(a) What is the most common number of pens and pencils that pupils carry to school? **[2]**

(b) What is the average number of pens and pencils that a pupil carries to school? **[2]**

14. Mega Shoe Store caters especially to men with big feet. The sales figures for a particular week are as given in the table below.

Size	9	10	11	12	13	14
Number of pairs sold	20	6	15	28	7	4

(a) Draw a histogram to illustrate the information.

(b) **(i)** Find the total number of pairs of shoes sold.

(ii) Given that the average price of a pair of shoes sold is \$65, find the shop's takings for the week.

(c) Find the mode, median and mean size of shoes sold. **[7]**

End-of-Year Specimen Paper

Answer **all** *the questions.* **Time: $2\frac{1}{2}$ h**

1. **(a)** Factorise $a^2 - b^2$. **[1]**

(b) Use the result obtained in **(a)** to find the value of x where $20x = 402^2 - 398^2$. **[2]**

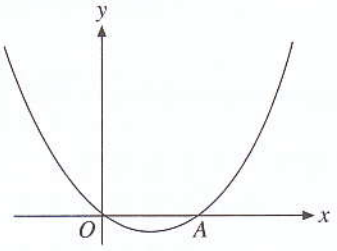
2. Given that $x + y = -4$ and $x - y = 8$, find the value of $y^2 - x^2$. **[2]**

3. The diagram shows the graph of $y = x(x - 4)$. The graph passes through the origin O and the point A.

(a) Find the coordinates of A. **[1]**

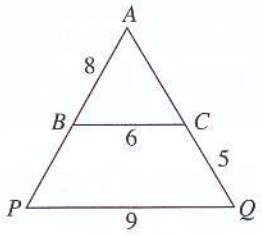
(b) Write down the equation of the axis of symmetry of the graph. **[1]**

(c) Find the smallest possible value of y and the value of x when this occurs. **[2]**



4. In the figure, not drawn to scale, triangles ABC and APQ are similar. Given that $AB = 8$ cm, $BC = 6$ cm, $CQ = 5$ cm and $PQ = 9$ cm, calculate

(a) BP, **(b)** AC. **[4]**



5. Given that $y = \dfrac{5x + 3}{x - 5}$, express x in terms of y. **[3]**

6. The distance between two cities, P and Q on a map drawn to the scale of 1 : 50 000, is 16 cm. Find the distance of PQ on another map drawn to a scale of 1 : 60 000. **[2]**

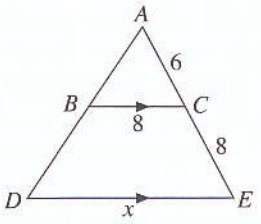
7. A solid plasticine cone of base radius 6 cm and height 3 cm is formed into a solid sphere of radius r cm. Calculate r. **[2]**

8. Solve the simultaneous equations:
$5x + 3y = 23$, $7y - x = -35$. **[3]**

9. In the figure, BC is parallel to DE.

(a) Name a pair of similar triangles. **[1]**

(b) Given that $AC = 6$ cm, $CE = 8$ cm, $BC = 8$ cm and $DE = x$ cm, calculate the value of x. **[2]**



10. Given that $\dfrac{p + 5q}{2p - q} = \dfrac{9}{5}$, find the value of $\dfrac{2p}{q}$. **[3]**

11. The lengths of the diagonals of a rhombus are 10 cm and 24 cm. Find the perimeter of the rhombus. **[3]**

12. If $\varepsilon = \{a, e, i, o, u, z\}$, $A = \{a, e, i\}$, $B = \{i, o\}$ and $C = \{o, u, z\}$.

(a) List the members of

(i) $A \cap B$,

(ii) $A \cup C$.

(b) Find the value of

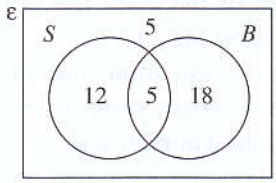
(i) $n(A \cap B')$,

(ii) $n(A \cup B)$. **[4]**

13. Given that y is directly proportional to the square of $(x + 1)$ and that the difference between the values of y when $x = 1$ and $x = 2$ is 20, calculate the value of y when $x = 3$. **[3]**

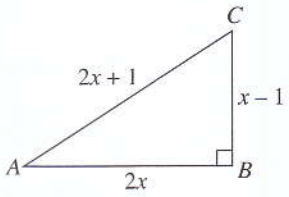
14. A box contains 5 blue balls, 6 green balls and 9 white balls. A ball is selected at random. Find the probability of selecting
(a) a green ball,
(b) a blue or a black ball,
(c) a ball that is not yellow. **[3]**

15. The Venn diagram shows the number of pupils of a class in the swimming team and basketball team. A pupil is selected at random from the class. Find the probability that the pupil is
(a) in the swimming team,
(b) in the basketball team only,
(c) not in the swimming and basketball teams,
(d) not in the basketball team. **[4]**



16. In an independent school, all 300 Secondary One students study either Computer Science, or a third language or both Computer Science and a third language. 45% of these students study Computer Science and 65% study a third language. By drawing a Venn diagram, or otherwise, find the number of students who study
(a) both Computer Science and a third language, **[2]**
(b) Computer Science only. **[1]**

17. In the right-angled triangle ABC, $AB = 2x$ cm, $BC = (x - 1)$ cm and $AC = (2x + 1)$ cm. Form an equation in x and hence find
(a) x, **[3]**
(b) the area of the triangle. **[1]**



18. Using a scale of 2 cm to 1 unit on each axis, draw the graphs of $x + 2y = 10$ and $y = 1.5x + 1$ for values of x from -1 to 3. Use your graph to solve the simultaneous equations $x + 2y = 10$ and $y = 1.5x + 1$. **[4]**

19. Two variables c and t are such that $c = at^3 + \frac{b}{t^2}$. Given that $c = 74$ when $t = 1$, and $c = 34$ when $t = 2$, find the value of c when $t = 3$. **[4]**

20. A sphere has a radius of 4.6 cm. Calculate, giving your answer correct to 2 decimal places, its
(a) surface area, **[2]**
(b) volume, taking $\pi = 3.142$. **[3]**

21. A pyramid stands on a rectangular base measuring 15 cm along one side. If its height is 18 cm and its volume is 826 cm^3, find the length of the other side of its base. **[3]**

22. Two numbers, x and y, such that the sum of their squares is equal to twice their product plus 64.
(a) Express the condition given in equation form. **[2]**
(b) Use the answer obtained for **(a)** to find the difference between the two numbers. **[2]**

23. The following is an incomplete table of values for the graph of $y = (1 - 2x)(3 + x)$.

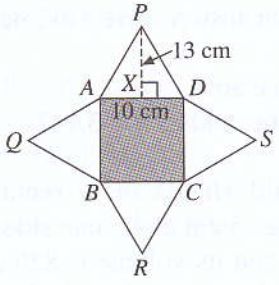
x	−4	−3	−2	−1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y	−9	0		6			0		−15

Copy the table, calculate and write down the missing values of y. Taking 2 cm to represent 1 unit on the x-axis and 2 cm to represent 2 units on the y-axis, draw the graph of $y = (1 - 2x)(3 + x)$ for $-4 \le x \le 2$. **[3]**
Use your graph to write down
(a) the greatest value of y,
(b) the value of y when $x = -2.3$,
(c) the values of x when $y = -3$. **[4]**

24. In the diagram, $ABCD$ is a square of side 10 cm and the triangles are all isosceles and identical. In the triangle PAD, PX is drawn perpendicular to AD and is of length 13 cm.
The shape in the diagram can be folded along the edges of the square to form a right pyramid with $ABCD$ as base, and with the four points P, Q, R and S coming together at the vertex. Calculate

(a) the total surface area of the pyramid formed, **[2]**

(b) the volume of the pyramid. **[3]**



25. A six-sided die is thrown 39 times. The scores are shown in the table below.

Score	1	2	3	4	5	6
Number of times	7	9	6	4	5	8

(a) From these results, write down

(i) the mode, and **[1]**

(ii) the median **[2]**

(b) Calculate the mean score, giving your answer correct to 1 decimal place. **[2]**

(c) Construct a histogram for the results above. **[3]**

26. The cost of printing copies of a book is given by the equation

$$c = a + \frac{b}{n},$$

where c dollars is the cost per copy, n is the number of copies printed and a and b are constants. When 300 copies of the book are printed, the cost per book is \$8.50 and when 700 copies of the book are printed, the cost per book is \$4.50.

(a) Form two equations in a and b.

(b) Solve these equations to find the value of a and the value of b.

(c) Calculate the cost per copy when 200 copies of the book are printed.

(d) How many copies of the book should be printed if the cost per book is to be \$5.70? **[7]**

Answers

(Practice Questions, Tests and Specimen Papers)

Chapter 1

1. (a) QP (b) PC
(c) CA (d) $Q\hat{P}C$
(e) $C\hat{A}B$ (f) $B\hat{C}A$

2. $AB = CD$, $BD = DB$, $AD = CB$, $A\hat{B}D = C\hat{D}B$, $A\hat{D}B = C\hat{B}D$, $B\hat{A}D = D\hat{C}B$

3. $PQ = QP$, $QS = PR$, $PS = QR$, $P\hat{R}Q = Q\hat{S}P$, $R\hat{P}Q = S\hat{Q}P$, $P\hat{Q}R = Q\hat{P}S$

4. $AB = AC$, $BQ = CP$, $AQ = AP$, $A\hat{B}Q = A\hat{C}P$, $A\hat{P}C = A\hat{Q}B$, $B\hat{A}Q = C\hat{A}P$

5. (a) $p = x = 40$, $q = 92$, $c = 48°$, $z = 7$
(b) $q = 8.5$, $y = 32$, $a = 58$, $b = 10$
(c) $p = 6$, $r = 75$, $a = 39$, $q = 66$
(d) $p = 7$, $q = 6$, $a = 6.5$

6. (a) $p = 73$, $q = 6$, $b = 102$, $s = 7$
(b) $a = 11.5$, $x = 41$, $c = 42$, $d = 62$, $y = 11$
(c) $x = 7.2$, $b = 7$, $a = 5.8$, $s = 83$
(d) $p = 92$, $a = 7.6$, $b = 8.0$, $r = 57$

7. (a) $x = 40$, $y = z = 50$
(b) $x = 44$, $y = 54$, $z = 82$
(c) $x = 6.75$, $y = 88$
(d) $x = 6.3$

8. (a) $x = 10\frac{4}{5}$, $y = 11\frac{1}{5}$
(b) $x = 8\frac{3}{4}$, $y = 10\frac{2}{3}$
(c) $x = 8\frac{1}{3}$, $y = 5\frac{2}{5}$
(d) $x = 18$, $y = 10\frac{2}{3}$
(e) $x = 6\frac{2}{3}$, $y = 7\frac{11}{15}$
(f) $x = 10$, $y = 22\frac{1}{2}$
(g) $x = 9$, $y = 8.4$
(h) $x = 14\frac{1}{7}$, $y = 12\frac{4}{7}$

9. (a) $4\frac{1}{2}$ cm (b) $5\frac{1}{7}$ cm

10. (a) 12 (b) $13\frac{1}{2}$

11. (a) 36 km (b) 15.75 km
(c) 21.3 km (d) 3.9 km

12. (a) 38.4 cm (b) 32 cm
(c) 172.8 cm (d) 2 cm

13. (a) 24 km (b) 40 km
(c) 60 km (d) 8 km

14. 1 cm to 7.5 m

15. 2.8 km, 4.2 cm

16. (a) 4.5 km
(b) 4.125 km
(c) 16.25 km
(d) 1.85 km

17. (a) 2 cm (b) 0.5 cm
(c) 9 cm (d) 6 cm

18. (a) 1 : 450 000
(b) 1 : 150 000
(c) 1 : 80 000
(d) 1 : 1 400 000

19. 88 km

20. (a) 900 km
(b) 600 km
(c) 1537.5 km
(d) 1650 km

21. 80 cm²

22. (a) 0.2 km² (b) 0.72 km²
(c) 3 km² (d) 4.96 km²

23. (a) 270 m (b) 4500 m²

24. 1 : 500, 64 m, 240 m²

25. 4 cm², 16 cm²

26. (a) 7.875 km
(b) 1 : 75 000
(c) 57.6 cm²

27. (a) 1350 m²
(b) 937.5 m²
(c) 540 000 m²
(d) 153 600 m²

28. (a) 9 cm (b) 24 cm
(c) 100 cm (d) 245 cm

29. (a) 144 cm² (b) 576 cm²
(c) 16 cm² (d) 2.25 cm²

30. (a) 6.48 km (b) 8.4 cm
(c) 5.184 km²

31. 1 : 250 000, 9 cm²

32. 400 cm

33. (a) T (b) F (c) T
(d) F (e) T (f) F
(g) F (h) F (a) T
(j) T (k) F (l) F

34. 6.4 m

35. (a) 5.04 cm (b) 12.67 cm

36. (a) $13\frac{1}{5}$ (b) 35 cm

37. (a) 27 cm (b) 48 cm

Chapter 2

1. \$31.05, 44.4 litres

2. 672 **3.** 108

4. 90 kg **5.** 82

6. 693 kg **7.** 112

8. 24 kg; 106.8 kg

9. 112, 96, 120

10. 87 : 89

11. (a) 3 (b) 40

12. (a) 187.5 (b) 39

13. 18, 60, 90

14. (a) $4\frac{1}{2}$ (b) ±8

15. (a) 64 (b) $2\frac{1}{2}$

16. (a) $\frac{1}{3}$ (b) $p = 0$
(c) $q = 99$

17. (a) 7 (b) 8

18. 14

19. (a) ±4 (b) 18

20. (a) 4 (b) $\frac{2}{9}$

21. $62\frac{1}{2}$ **22.** 6

23. 20 **24.** 2

25. $\frac{1}{2}$ **26.** 128

27. $F = \frac{6}{25}\ m$

(a) 72 (b) 425

28. $I = \frac{11}{200}\ P$, \$88

29. (a) 245 m (b) 2 s

30. $18\frac{3}{4}$ cm

31. (a) $d = 6t^2$ (b) 600 m

32. $w = 6t$, 15 g

33. (a) 0.686 litre

(b) 24 cm

34. 4500 **35.** 77

36. (a) 567 joules

(b) 3 cm

37. (a) 12 cm (b) $2\frac{1}{4}$ cm

38. (a) 18 m/s (b) 196 m

39. 8820 newton/m^2

40. (a) 5445 joules

(b) 2.5 amperes

41. (a) 4.8 s (b) 25 cm

42. 56 cm **43.** 200 N/m^2

44. (a) 6.25×10^2 kc/s

(b) 3.125×10^2 m

45. $31\frac{1}{4}$ ohms **46.** 4

47. 405

48. (a) $\frac{22}{7}$ (b) $3\frac{1}{7}$

(c) d is directly proportional to the square root of A

49. 3.4 kg, 9129.4 km

50. $F = \frac{k}{R^2}$,

(a) 51 200 (b) 10

51. $t = \frac{400}{v}$, 75 km/h

52. (a) $a = 25$, $b = 50\ 000$

(b) (i) \$60 500

(ii) \$81 250

53. (a) \$65 (b) \$65 000

54. 292

55. 13

Chapter 3

1. (a) $6x + 21y$

(b) $12h - 20k$

(c) $-8a - 12b$

(d) $18x - 42y$

(e) $10h + 45k$

(f) $-35h - 49k$

(g) $-32p + 24q$

(h) $-18h + 27k$

2. (a) $10x^2 + 15xy$

(b) $-6xy + 24x^2$

(c) $6m^2 + 3mn$

(d) $-8hk - 12h^2$

(e) $-36a^2 - 63ab$

(f) $-8xy - 20y^2$

(g) $21x^2 - 28xy$

(h) $40p^2 - 16pq$

3. (a) $11x + 18$

(b) $p + 38$

(c) $3x - 9$

(d) $28x - 59$

(e) $47x + 121$

(f) $22p - 28$

(g) $28a - 26$

(h) $-14x + 57$

4. (a) $11x^2 + 6x$

(b) $9x^2 - 5x$

(c) $14x^2 - 14xy$

(d) $31p^2 + 19pq$

(e) $19a^2 - 2ab$

(f) $5x^2 + 33xy$

(g) $-12x^2 - 9xy$

(h) $14pq - 14p^2$

5. (a) $x^2 + 12x + 35$

(b) $x^2 + 4x - 77$

(c) $28 - x - 2x^2$

(d) $2x^3 - 4x^2 + 6x - 12$

(e) $2x^3 + 3x^2 - 8x - 12$

(f) $2x^2 - 7xy + 6y^2$

(g) $12x^2 + x - 20$

(h) $8x^2 + 22xy - 21y^2$

(i) $20x^2 + 53xy + 35y^2$

(j) $16x^2 - 9$

(k) $4x^2 - 10cx - 6bx + 15bc$

(l) $(ab)^2 + 3ab - 40$

(m) $x^3 - 2x^2 - 5x + 6$

(n) $x^4 - 16$

(o) $x^8 - y^4$

6. (a) $9x^2 + 6xy + y^2$

(b) $36x^2 + 60xy + 25y^2$

(c) $x^2 + 4 + \frac{4}{x^2}$

(d) $x^2 + \frac{2}{3}xy + \frac{y^2}{9}$

(e) $9x^2 + \frac{3xy}{2} + \frac{y^2}{16}$

(f) $49x^2 - 14xy + y^2$

(g) $25x^2 - 90xy + 81y^2$

(h) $x^2y^2 + 4xy + 4$

(i) $x^4 + 6x^2 + 9$

(j) $x^4y^2 + 2x^2yz + z^2$

(k) $a^2b^2c^2 - 2abcx + x^2$

(l) $x^6 + 8x^3 + 16$

(m) $\frac{a^2}{b^2} + \frac{2a}{bc} + \frac{1}{c^2}$

(n) $\frac{4}{x^2} + \frac{12}{xy} + \frac{9}{y^2}$

(o) $\frac{a^2}{b^2c^2} - \frac{6a}{bc} + 9$

(p) $\frac{a^2}{b^2} + 2 + \frac{b^2}{a^2}$

(q) $\frac{x^4}{y^2} - 2x + \frac{y^2}{x^2}$

(r) $\frac{a^2}{b^2} + 6a + 9b^2$

7. (a) $27 - a^3$

(b) $x^3 + y^3$

(c) $6a^2 - 5ab - 4b^2 + 2ac + bc$

(d) $2x^3 - 5x^2 - 11x - 4$

(e) $x^4 - 2x^3 - 3x^2 + 8x - 4$

(f) $3a^3 + a^2 - 4a + 12$

(g) $2a^3 + a^2 - 19a + 10$

(h) $-5a^3 + 37a^2 - 15a + 7$

(i) $2p^2 - pq - 6q^2 - p + 2q$

(j) $x^4 - 1$

(k) $x^3 - y^3$

(l) $a^4 - 4a^3 + 6a^2 - 4a + 1$

8. (a) $x^2 - xy - 4y^2$

(b) $4y^2$

(c) $10x^2 - 28x - 17$

(d) $5x^2 - 16x - 3$

(e) $29a - 13a^2 - 18$

(f) $-x^3 - 2x^2 - 10x - 17$

(g) $-x^2 - 2xy - 5y^2$

(h) $4xy - 6x^2y$

9. (a) $8(3x + 2)$

(b) $2ab(1 + 2c)$

(c) $abc(1 - ac^2)$
(d) $x(2 + k)$
(e) no factor
(f) $2ab^2c(ab - 4c^2)$
(g) $pq(p - 2q + 4pq)$
(h) $ab(a - b + ab)$
(i) $2x(1 - 2x + 4y^2)$
(j) $a^2x(5 - 3ax + 6x)$
(k) $2a^2(3 + 4a - 5a^3)$
(l) $3xy(4x^2 - 3xy + 2y^2)$

10. (a) $(a + c)(b - a)$
(b) $(x + y)(x + 3z)$
(c) $(x^2 + 4)(z - y)$
(d) $(2b - 5d)(4a - 3c)$
(e) $(2a + 3b)(x - 2y)$
(f) $(x^2 + y)(x - 3y)$
(g) $(a + 1)(a - 1 + b)$
(h) $(x + 2)(3y - 5)$
(i) $(a - b)(a + 3c)$
(j) $(y + 2)(x^2 - 3)$
(k) $(x - 4)(x^2 + 1)$
(l) $(x - 3)(x^2 + 4)$
(m) $(x - y)(1 - x - y)$
(n) $(2x + y)(2x - y + 3)$
(o) $(x + 5)(x - 1 + a)$
(p) $(x^3 + 2)(x + 3y)$
(q) $(a^2 - 3b)(x + 4y)$
(r) $(5x - 4y)(x + z)$

11. (a) $\left(\frac{1}{2}x - y\right)\left(\frac{1}{2}x + y\right)$
(b) $(3a - x)(3a + x)$
(c) $(2b + b)^2$
(d) $(3a - b)^2$
(e) $4b(a + b)$
(f) $(x^2 + 4)^2$
(g) $(ab - 5)^2$
(h) $(5x - 2)^2$
(i) $\left(\frac{1}{2}x - \frac{1}{4}y\right)^2$
(j) $(3a + 2)^2$
(k) $(4a + 5b)^2$
(l) $(7a - 2b)^2$
(m) $(2x + 9)(2x - 9)$
(n) $a(9b + 2c)(9b - 2c)$
(o) $(x^2 + 9y^2)(x + 3y)(x - 3y)$

12. (a) $(x + 5)(x - 7)$
(b) $2(x - 3)(x + 7)$
(c) $(3x + 1)(x - 2)$
(d) $(2x + 1)(x - 3)$
(e) $(x + 15)(x + 5)$
(f) $(x - 4)(x - 7)$
(g) $(x - 7)(x + 11)$
(h) $(x - 11)(x + 14)$
(i) $(x - 4)(x - 17)$
(j) $(x + 9)(x - 19)$
(k) $(12x + 5)(x - 3)$
(l) $(5x - 1)(3x + 1)$
(m) $3(x - 6)^2$
(n) $(3x - 4)(x + 5)$
(o) $(3x + 5)(x - 2)$

13. (a) $x(2x + 7)(2x - 7)$
(b) $5(x + 2)(x - 2)$
(c) $3a(3a + 4)(3a - 4)$
(d) $(3 + a - b)(3 - a + b)$
(e) $(x - 3 + 4y)(x - 3 - 4y)$
(f) $2x(3x + 2y)(3x - 2y)$
(g) $x^2(x + 5)(x - 5)$
(h) $(7 + x)(7 - x)$
(i) $4xy(x - 2y)$
(j) $(2x - p + 2)(2x + p - 2)$
(k) $3(x + 2y)(x - 2y)$
(l) $5(x + y)(x - y)$
(m) $x(2x - 1)(x + 2)$
(n) $(6x + 5y)(x - 2y)$
(o) $(4x - y)(2x - y)$
(p) $4y(3x - y)$
(q) $(t^2 + 2)(t + 2)(t - 2)$
(r) $(5x - 4y)(x + 4y)$
(s) $x(3x + 7y)(2x - 5y)$
(t) $x^3y^3(9x + 11y)(9x - 11y)$

14. (a) 9999 (b) 64 016 001
(c) 359 991 (d) 30 600
(e) 9140 (f) 646 000
(g) 1.026 (h) 40 000
(i) 4900 (j) 2500
(k) 8100 (l) 29 632 000

15. $(3x + 17)(x + 3)$; 317, 103
16. 60
17. 8114
18. (a) 67 (b) 76
19. 9
20. $(x + 3)(4x + 1)$; 103, 401
21. (a) –36 (b) 164
22. 55
23. 12 033
24. (a) 0 or 5 (b) 0 or $\frac{2}{3}$
(c) 0 or $\frac{2}{3}$ (d) 0 or $1\frac{2}{5}$
(e) $3\frac{1}{2}$ or 5
(f) $-1\frac{4}{5}$ or $2\frac{2}{3}$
(g) $p = \frac{5}{7}$, $q = \frac{2}{9}$
(h) $h = 1\frac{1}{5}$, $k = -1\frac{4}{11}$

25. (a) 0 or $-2\frac{1}{2}$
(b) 0 or $\frac{7}{8}$
(c) 0 or $3\frac{1}{2}$
(d) 1 or –5
(e) 8 or –2
(f) 1 or –7
(g) 1 or $-2\frac{1}{2}$
(h) 2 or 11
(i) 1 or –1
(j) 4 or –4

26. (a) $\frac{1}{2}$ or –4
(b) $-\frac{2}{5}$ or –3
(c) –2 or $3\frac{1}{2}$
(d) $1\frac{1}{3}$ or $-\frac{3}{4}$
(e) $\frac{3}{4}$ or $-\frac{2}{3}$
(f) $-1\frac{1}{2}$ or $1\frac{2}{3}$
(g) $-1\frac{1}{3}$ or $1\frac{2}{3}$
(h) $4\frac{1}{2}$ or $-1\frac{3}{4}$
(i) $\frac{1}{4}$ or $-1\frac{1}{2}$
(j) $\frac{2}{5}$ or $-1\frac{2}{3}$
(k) 4 or $-3\frac{1}{3}$

(l) -1 or $\frac{3}{7}$

(m) 12 or -2

(n) -4 or $\frac{2}{5}$

(o) 2 or 6

(p) $\frac{1}{2}$ or -3

(q) $-\frac{1}{2}$, 3 or 0

(r) 0, $\frac{1}{2}$, $-\frac{1}{3}$

(s) $\frac{2}{3}$ or $-\frac{1}{2}$

(t) -1 or $1\frac{2}{3}$

27. (a) $1\frac{5}{11}$

(b) $\frac{1}{4}$ or 3

(c) $-\frac{1}{6}$

(d) $1\frac{1}{3}$ or $-1\frac{1}{2}$

(e) 9 or -12

(f) 3

(g) 5 or $1\frac{1}{9}$

(h) 4 or -5

(i) 5 or $-1\frac{1}{2}$

(j) 2 or -1

(k) $-1\frac{3}{5}$

(l) 2

28. $p = -1$, $x = -2\frac{1}{2}$

29. 7, 9

30. 18 m, 86 m

31. 11, 14 or -11, -14

32. (a) $x = 3$ (b) 60 cm

33. 4 **34.** 12, 14

35. $x = 4$ **36.** 10

37. 13 yrs

38. $x = 72$; 10 h 25 min

39. $\frac{300}{x} - \frac{300}{x+3} = 5$; $x = 12$

Term I Revision Test

1. (a) $33\frac{1}{3}$ min

(b) 32 h

2. (a) 20 cm (b) 34.5 km

(c) 4.5 km^2 (d) 27 cm^2

3. (a) $x^3 + 4x^2 + 5x + 6$

(b) $8p^2 - 2pq - 5q^2$

(c) $\frac{1}{a(b+3)}$

4. (a) $2(x^2 + 4yz)(x^2 - 4yz)$

(b) $(8mn - 1)^2$

(c) $(a - 2b)(a + 2b + 3)$

5. (a) 18 (b) 1 280 000 m^2

6. (a) $\frac{560}{x}$ (c) \$7, 80

7. (a) -3 (b) $\frac{-2}{19}$

8. (a) 18 cm

(b) 2.5 km^2, 10 cm^2

9. $x = 5\frac{5}{7}$, $y = 17\frac{1}{2}$

10. (a) 100 (b) $H = \frac{5}{8}$; $x = 3$

11. (a) 32 cm (b) 187 500 ha

12. (a) $\triangle ADO$ (b) $\triangle COB$

(c) $\triangle ADC$

13. $x = 24$, $y = 24$

Chapter 4

1. (a) $15xy$ (b) $\frac{5x^6}{y}$

(c) $\frac{8b^2c^2}{3a^2}$ (d) $\frac{1}{2xy^3}$

(e) $\frac{xy^2}{3z^3}$ (f) $\frac{1}{3y^2z^2}$

(g) $\frac{z}{2x}$ (h) $\frac{2x^3y^2}{z^3}$

(i) $\frac{7x^4yz^3}{3}$ (j) $2a^2bc^8$

(k) $-3x^4y^7$ (l) $\frac{-4b^4c^{14}}{a}$

(m) $5x^8y^9$ (n) $\frac{-a^{10}bc^3}{z^2}$

(o) $\frac{4b^8}{ac^2}$ (p) $\frac{a^2b}{2(x+y)}$

(q) $\frac{3x^2(a-b)^2}{7y}$

(r) $\frac{x^2(a+b)}{3y(a-b)}$

(s) $\frac{3}{5a^3}$

(t) $\frac{ab^3}{3}$

(u) $\frac{3a^{n+1}b^2}{8}$

(v) $7a^{n-3}b^{n-1}$

2. (a) $\frac{2(k-2h)}{3h}$

(b) not possible

(c) $\frac{x}{y}$ (d) $\frac{-a}{b}$

(e) not possible

(f) $\frac{2}{3}$

(g) $\frac{a+b}{a-b}$ (h) $\frac{x}{x+4}$

(i) $\frac{b}{a-b}$ (j) $\frac{8a}{5}$

(k) not possible

(l) $\frac{2x+y}{6x+y}$ (m) $\frac{x+3}{x-7}$

(n) $\frac{5}{3x-4}$ (o) $\frac{2a-5}{2a+5}$

(p) $\frac{a+b-c}{b+c-a}$ (q) $\frac{3-4a}{7+3a}$

(r) $\frac{x+y}{x-5y}$ (s) $\frac{2x-3y}{3x}$

(t) $\frac{2a+b}{x-3y}$ (u) $\frac{x-2z}{x-z}$

(v) $\frac{3z}{3x-z}$

(w) $\frac{3x+y-2z}{5x}$

3. (a) $\frac{10y^2a^2}{3xzc}$ (b) $\frac{9b^2c^2}{4a^2}$

(c) $\frac{3y^5}{2xz^4}$ (d) $\frac{2b^4c}{a}$

(e) $\frac{x^4y}{z^2}$ (f) $\frac{15abx^2y}{16}$

(g) $\frac{16ac}{9}$ (h) $\frac{4xy^2a^2}{3c}$

(i) $\frac{5x^4z}{7y}$ (j) $\frac{7x}{8yz^2}$

(k) $\frac{27a^2by^2}{14}$ (l) $\frac{xz^2}{y^2}$

4. (a) $\frac{4a}{3b}$ (b) $\frac{8p}{q}$

(c) $\frac{3}{k^2h^3}$ (d) $\frac{ay^3}{2x^2}$

(e) $\frac{9a^2y^2}{16xz^3}$ (f) $\frac{nb^2}{8}$

(g) $\frac{36a^5b^2}{y}$ (h) $\frac{2pqsx}{3r^2y}$

(i) $\frac{9g}{28kl}$ (j) $\frac{27b}{56ac}$

(k) $\frac{x^3(a-1)}{a^2}$

(l) $\frac{1+x}{x}$

5. (a) 2 (b) a

(c) $3a$ (d) $5xy$

(e) $2mn$ (f) $11xy$

6. (a) $12ab$ (b) $15ab$

(c) $24abc$ (d) $15xy^2$

(e) $18abc$ (f) $40a^3b^3c^2$

7. (a) $\frac{3x+5}{4}$ (b) $\frac{6x+1}{6}$

(c) $\frac{9a-4}{10}$ (d) $\frac{17x}{12}$

(e) $\frac{23x-2}{6}$ (f) $\frac{17-11x}{12}$

(g) $\frac{x+1}{6}$ (h) $\frac{11-2x}{20}$

(i) $\frac{a+7}{18}$ (j) $\frac{13a-10}{12}$

(k) $\frac{29xy}{15a}$ (l) $\frac{13}{12x}$

(m) $\frac{51a-b}{30x}$ (n) $\frac{23x+34y}{12z}$

8. (a) $\frac{7x}{12}$ (b) $\frac{1}{6}(x+1)$

(c) $\frac{-13x-33}{30}$

(d) $\frac{7a}{2x+3}$

(e) $\frac{2a+15}{6(x-2y)}$

(f) $\frac{9-8x}{5x-4}$

9. (a) $\frac{1}{2x-3y}$ (b) $\frac{-2x-20}{x^2-16}$

(c) $\frac{4}{2a+1}$ (d) $\frac{a}{8+2c}$

(e) $\frac{ab}{2b+3a}$ (f) $\frac{2y+5x}{3y}$

10. (a) $\frac{7x+5}{(3x+1)(5x+3)}$

(b) $\frac{2}{x+2}$

(c) $\frac{xy}{x+y}$

(d) $\frac{3x-4}{2x(x-2)^2}$

(e) $\frac{2}{x^2-4}$

(f) $\frac{-1}{1+x}$

(g) $\frac{3(x+1)}{x^2+x-6}$

(h) $\frac{3t+4}{2(t-1)(t+2)}$

(i) $2(x+2)$

(j) $\frac{4x+3}{(x-1)(x-2)}$

(k) $\frac{-1}{2(x+1)}$

(l) $\frac{2}{x^2-4}$

(m) $\frac{x-3}{x-2}$

(n) $\frac{7x+6}{(x-2)(x-3)(x+3)}$

(o) $\frac{2x-6}{x^2-4}$

(p) $\frac{2}{(x-3)(x-4)}$

(q) $\frac{y^2}{(x-3y)(x+y)}$

(r) $\frac{x(3x+8)}{x^2-9}$

(s) $\frac{3x-19}{(x-1)(x-4)(x+3)}$

(t) $\frac{13-x}{(x+1)(x+3)(x-5)}$

(u) $\frac{2x-1}{x(x-1)(x-2)}$

11. (a) $5\frac{5}{6}$ (b) $1\frac{1}{6}$

(c) 8 (d) $5\frac{2}{3}$

(e) $3\frac{1}{7}$ (f) $\frac{4}{11}$

(g) $-\frac{1}{9}$ (h) 25

(i) $-3\frac{7}{8}$ (j) $-24\frac{1}{3}$

(k) $-\frac{4}{7}$ (l) $-\frac{25}{27}$

(m) -2 (n) -1

(o) $1\frac{1}{4}$ (p) $\frac{9}{13}$

(q) $1\frac{8}{23}$ (r) $\frac{5}{8}$

(s) 4 (t) $-10\frac{1}{35}$

(u) $1\frac{1}{2}$

12. (a) 1 or 7

(b) 1 or -3

(c) 25 or -12

(d) 5 or -1

(e) 1 or -1

(f) 1 or $\frac{1}{4}$

(g) -1

(h) 2

(i) 2 or $-3\frac{7}{9}$

(j) 1

(k) 0 or 2

(l) 2 or -2

13. (a) $-1\frac{1}{11}$ (b) $-1\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{17}{60}$

(e) -1 (f) $2\frac{1}{4}$

(g) $1\frac{1}{2}$ (h) $1\frac{3}{8}$

(i) $-7\frac{1}{4}$ (j) -30

(k) $\frac{13}{15}$ (l) 6 or $-1\frac{1}{3}$

(m) $\frac{1}{3}$ (n) 60 or -48

(o) $-1\frac{5}{7}$ (p) $1\frac{2}{3}$

(q) 30 or -20

(r) 3 or $\frac{1}{2}$

(s) $2\frac{1}{2}$ (t) 5

14. (a) $a = b - x$

(b) $a = k + h$

(c) $a = b + c + d$

(d) $a = d + e - c$

(e) $a = x - y$

(f) $a = z - 2k$

(g) $a = p + q$

(h) $a = p - 5k$

(i) $a = 7k - p$

(j) $a = k^2 + b + c$

(k) $a = b + k - h^3$

(l) $a = k - m - n$

(m) $a = m - n - h$

(n) $a = \dfrac{7k + h}{3}$

(o) $a = 5pq + q - p^2$

(p) $a = x^2y - 3xy$

(q) $a = 3$ (r) $a = \dfrac{3y}{x}$

(s) $a = \dfrac{2xy}{3k}$

(t) $a = \dfrac{p - q + k}{k}$

(u) $a = \dfrac{5y - 4}{x^2}$

15. (a) $a = \dfrac{b - c + k^3}{h}$

(b) $a = \dfrac{x^3 - y}{5y^2}$

(c) $a = \dfrac{A}{bx}$

(d) $a = \dfrac{5x}{7ky}$

(e) $a = \dfrac{4m(5b + c)}{k}$

(f) $a = \dfrac{3kby}{10x}$

(g) $a = \dfrac{t^2}{e(5n + 4)}$

(h) $a = \dfrac{y^3}{x - y^2}$

(i) $a = \dfrac{4m - mkx}{kx}$

(j) $a = \dfrac{1}{5}(7 + 5b)$

(k) $a = \dfrac{v - mc}{m}$

(l) $a = \dfrac{7y - k}{7b + 4y}$

(m) $a = \dfrac{3z - 5}{z - 2}$

(n) $a = \dfrac{7 + 4x}{x - 3}$

(o) $a = \dfrac{4xh + 4 + 4x^2y - 2x}{5bx - 3k}$

16. (a) $x = \dfrac{ap + q^2}{2q - a}$

(b) $a = \dfrac{11x}{3n - 3}$

(c) $b = \dfrac{c}{a - 2}$

(d) $x = \dfrac{11y}{3y + 1}$

(e) $p = \dfrac{4sT}{4r - T}$

(f) $u = \dfrac{2fv}{3v - f}$

(g) $y = \dfrac{4x - 12}{5 + x}$

(h) $b = \dfrac{5c}{4 + 2ac}$

(i) $y = \dfrac{3ax - abx}{b^2 + 4a}$

(j) $x = \dfrac{b - yb}{yc - a}$

(k) $f = \dfrac{uv}{1 + v}$

(l) $x = \dfrac{16}{y - 10}$

(m) $c = \dfrac{5F - 160}{9}$

(n) $k = \dfrac{ER - PR}{P}$

(o) $x = \dfrac{4k + 1}{2 - k}$

(p) $y = \dfrac{15xz}{8x - 10z}$

(q) $m = \dfrac{4k}{12x + 3y}$

(r) $y = \dfrac{5kx + 2x + 20}{2kx + 11}$

(s) $y = \dfrac{21b + 20a}{2}$

(t) $k = \dfrac{b - a}{h}$

(u) $b = \dfrac{2acd}{3ad + 4ac - cd}$

17. (a) $x = \sqrt{\dfrac{2 - 3y}{1 + 2y}}$

(b) $x = \dfrac{y^2 + 3y}{2}$

(c) $x = \dfrac{a^2}{(ay - 1)^2}$

(d) $x = \sqrt{\dfrac{2py^2 + 5k}{k}}$

(e) $x = \sqrt{\dfrac{kb}{2ah + k}}$

(f) $x = \dfrac{hk^2 + b^2}{b}$

(g) $x = -a$

(h) $x = \pm\dfrac{a}{b}\sqrt{b^2 - y^2}$

(i) $x = \dfrac{y(1 - mt)}{1 - mz}$

(j) $x = \dfrac{v^2 - 3gh}{g}$

18. (a) $y = \dfrac{-c - ax^2}{b}$

(b) $n = \dfrac{a - klm}{km}$

(c) $g = \dfrac{4\pi^2 d}{t^2}$

(d) $b = \dfrac{2abc}{4ac - 3ad - cd}$

(e) $h = \sqrt{\dfrac{A^2 + \pi^2 r^4}{\pi^2 r^2}}$

(f) $y = \dfrac{xgm}{gm + k^2}$

(g) $r = \sqrt{\dfrac{3v}{4\pi h}}$

(h) $t = k\sqrt{\dfrac{1 - 2x^2}{1 + 3x^2}}$

19. $m = \dfrac{u^2}{u - f}$

20. $x = \dfrac{qz - pq}{q - pz + p^2}$

21. 20, 25 **22.** 12, 21

23. 62, 63 **24.** 9

25. \$28 **26.** 20, 22, 24

27. 91, 93, 95 **28.** \$102 000

29. 5 yrs **30.** $10\frac{2}{3}$

31. 36 yrs **32.** $5\frac{1}{4}$ h

33. \$8

34. (a) $\frac{1}{20}$, $\frac{1}{12}$

(b) $\frac{2}{15}$

(c) $7\frac{1}{2}$ min

35. 56 **36.** 18*G*, 22*B*
37. 45 km **38.** 56
39. 48 **40.** 5
41. 3 h 36 min **42.** 48 km
43. 15 km/h

Chapter 5

1. (5, 2) **2.** (1, –2)
3. (6, –5) **4.** (–1, –2)
5. (6, –1) **6.** (0, 4)
7. (3, 2) **8.** (2, 5)
9. (4, –1) **10.** $\left(\frac{3}{5}, -\frac{3}{4}\right)$
11. $\left(5\frac{1}{2}, \frac{1}{2}\right)$ **12.** $\left(\frac{3}{4}, -\frac{3}{2}\right)$
13. (6, –3) **14.** (3, –2)
15. $\left(4, \frac{1}{2}\right)$ **16.** $\left(\frac{5}{16}, -1\frac{7}{8}\right)$
17. $\left(1\frac{5}{6}, \frac{3}{4}\right)$ **18.** (2, –1)
19. $\left(1, -1\frac{1}{2}\right)$ **20.** $\left(3\frac{5}{7}, 1\frac{4}{7}\right)$
21. (2, –1) **22.** $\left(\frac{1}{2}, -2\right)$
23. (3, 12) **24.** (1.2, –2.5)
25. (3.2, 4.5) **26.** (2, 4)
27. (2, –5) **28.** (7, 4)
29. $\left(-5\frac{1}{2}, 3\frac{5}{8}\right)$ **30.** (9, –3)
31. $\left(1\frac{1}{2}, 1\frac{1}{2}\right)$ **32.** $\left(\frac{1}{2}, \frac{1}{3}\right)$
33. $\left(\frac{2}{5}, 1\frac{2}{3}\right)$ **34.** $\left(-\frac{1}{2}, -\frac{2}{3}\right)$
35. $\left(\frac{6}{7}, \frac{5}{14}\right)$ **36.** $\left(\frac{3}{2}, -\frac{1}{2}\right)$
37. $\left(-\frac{5}{6}, -\frac{1}{3}\right)$ **38.** $\left(1\frac{2}{5}, -\frac{3}{5}\right)$
39. (10, 10) **40.** (1, 3)
41. (15, –6) **42.** (–1, –4)
43. (2, –3) **44.** $\left(2\frac{1}{3}, 1\frac{1}{2}\right)$
45. $\left(\frac{1}{2}, -1\frac{1}{3}\right)$ **46.** $\left(2\frac{2}{3}, 1\frac{1}{5}\right)$
47. $\left(1\frac{3}{5}, 2\frac{1}{5}\right)$ **48.** (–3, 6)
49. (–1, 8) **50.** (–1, 2)
51. (12, –4) **52.** (4, 2)
53. (3, 4) **54.** (4, 3)
55. (10.5, –3) **56.** $\left(2\frac{1}{2}, \frac{1}{2}\right)$
57. (–4, –3) **58.** (5, 2)
59. (2, –1) **60.** (10, –4)
61. (35, –28) **62.** 2 : 1
63. $a = 1$, $b = -12$
64. $a = -3$, $b = 1$
65. 24, 8 **66.** 52.5°
67. $\frac{13}{17}$ **68.** $\frac{3}{4}$, $\frac{3}{8}$
69. 16, 39
70. 32 km/h; 40 km/h
71. 240 km **72.** 17, 6
73. $\frac{7}{9}$ **74.** \$23, \$6
75. \$12, \$5 **76.** 30, 15
77. \$25.20 **78.** \$22.40
79. \$5.80 **80.** 13, 12
81. 25¢, 30¢ **82.** 300, 240
83. \$80, \$60 **84.** 10 yrs
85. 16 yrs, 50 yrs
86. 32 yrs **87.** 42 yrs, 14 yrs
88. 16 yrs **89.** 8 yrs
90. 252 m², 66 m
91. 391 m², 80 m
92. 220 m², 64 m
93. 867 m², 136 m
94. 30 m **95.** 32.4 m
96. 27 m **97.** 21 m
98. 36 **99.** 5 m/s, 3 m/s
100. 48

Chapter 6

1. **(a)** 13.7 **(b)** 5.24
(c) 16.7 **(d)** 16.6
(e) 24.7 **(f)** 20.8
(g) 7.94 **(h)** 45.6
2. **(a)** 13, 17.2 **(b)** 40, 21
(c) 11.2, 21.1
(d) 12.0, 18.4
(e) 7.55, 9.06
(f) 4.11, 22.7
3. 18.4 cm **4.** 72 cm²
5. 62.2 cm **6.** 26.46 cm
7. 51 km **8.** 8.94 cm
9. 6.93 cm; 27.7 cm²
10. 278 km **11.** 3.30 m
12. 9.66 cm **13.** 13 cm, 4 cm
14. 14.1 m **15.** 35.8 km
16. $x = 29.28$ **17.** 30.41 cm
18. 22.4 cm **19.** 120 cm²
20. 40.0 cm
21. **(a)** 15.2 cm **(b)** 17.2 cm
22. **(a)** 8.89 cm **(b)** 12.86 cm
(c) 116.9 cm²
23. 31.2 cm **24.** 650 cm²
25. $x = 9.04$ **26.** $x = 11.36$
27. $x = 4.80$ **28.** $x = 6$
29. $x = 6$; 24 cm²
30. $x = 4$; 40 cm

Term II Revision Test

1. **(a)** $(8s + 1)(3s - 2)$
(b) $(8a - 5b)(8a + 5b - 1)$
2. **(a)** 3 **(b)** 7 or –3
3. **(a)** $r = \frac{5(2p - q)}{q - p - 1}$; $-7\frac{1}{2}$
(b) $x = 3$, $y = 5$
4. **(a)** $\frac{3y + 2}{5}$ **(b)** $\frac{u(k - v)}{(u - k)(u - v)}$
(c) $\frac{b - 2}{b - 1}$
5. **(a)** $3 + x$ **(b)** $\frac{2y(y - 3)}{y + 3}$
(c) $\frac{5u + v}{u}$
6. **(a)** $\frac{1}{10}$ **(b)** 1 or $-\frac{1}{3}$
7. **(a)** 9 cm **(b)** 15.59 cm
8. **(a)** $t = \frac{x^2 - 2a}{b}$; –4
(b) **(i)** $\frac{3}{3k + 5}$ **(ii)** $3x - 1$
9. **(a)** $a + 729b = 281$
$a + 1296b = 344$
(b) $a = 200$, $b = \frac{1}{9}$
(c) **(i)** 641 newtons
(ii) 45 km/h
10. **(a)** 12 cm, 30 cm²
(b) 27
11. **(a)** 12 cm **(b)** 15 cm
(c) 54 cm²

Mid-Year Specimen Paper

1. 12 km/h
2. (a) 0 or 3 (b) $\frac{1}{3}$ or $-\frac{1}{2}$
3. (a) $5(x+2)(x-2)$
   (b) $(a-b)(2c+d)$
4. 489
5. 4.5 cm
6. $x = 5.4$, $y = 3.2$
7. $x = \frac{y+b}{3-a}$
8. $253\frac{1}{8}$ cm^2
9. (a) $8xy - 3y^2$
   (b) $\frac{18y+12x}{16y^2-9x^2}$
10. $17\frac{9}{23}$ ohms
11. 8.15 m
12. 42.2 m
13. (a) $\frac{x+2}{6}$ (b) $\frac{5y-7}{xy}$
14. $\triangle ABD$ and $\triangle BCD$
    $CD = \frac{ax}{c}$
15. (a) 15
    (b) $B = \frac{1}{2}(P - 2L)$
16. (a) $(a+b)(a-b)$
    (b) 7748
17. 80 cents, \$1.50
18. $\frac{2x+5}{2x}$
19. $2x^2 - x - 91 = 0$
    $x = 7$; 16.2 cm
20. $x = \frac{-3y-13}{2y+7}$
21. (a) 1 : 125 000
    (b) 11.75 km
    (c) 40.96 cm^2
22. (a) $(a-c)(2a-p)$
    (b) $1\frac{1}{2}$ or -4
23. $\frac{6}{17}$
24. $\triangle RUN$
25. (a) $1\frac{8}{17}$ (b) $x = 3$, $y = -2$
    (c) $\frac{2}{13}$ or $\frac{4}{5}$
26. (a) 25 chairs, 15 stools
    (b) The two equations represent two parallel lines which do not meet.
27. $\frac{480}{x}$, $\frac{480}{x-2}$; 40
28. 1521 cm^2
29. (a) $6a + b = 36$,
    $3a + b = 30$
    (b) $a = 2$, $b = 24$
    (c) (i) 50 (ii) $-\frac{1}{12}$

Chapter 7

1. 2304 cm^3
2. 360 cm^3
3. 45 m^3
4. 86.4 m^3
5. $15\frac{3}{4}$ cm
6. $10\frac{3}{4}$ m
7. 16 cm
8. $4\frac{2}{3}$ cm
9. (a) 302 cm^3
   (b) 302 cm^2
10. (a) 1357 cm^3
    (b) 1018 cm^2
11. (a) 4344 cm^3
    (b) 1629 cm^2
12. (a) 6033 cm^3
    (b) 3770 cm^2
13. (a) 5864 cm^3
    (b) 2036 cm^2
14. 4.77
15. 9.36
16. 2.91
17. 2.90
18. 6.4
19. 7.8
20. 5.3
21. 660 cm^2, 1276 cm^2
22. (a) 10 (b) 370 cm^2
23. 2.3 m^3, 5 bags
24. 72.66 cm^3
25. 817 cm^3, 423 cm^2
26. 8380 cm^3, 1995 cm^2
27. 59 373 mm^3, 7360 mm^2
28. 1023 m^3, 491 m^2
29. 3.50 cm, 180 cm^3
30. 7.00 mm, 1438 mm^3
31. 10.5 m, 4852 m^3
32. 3.00 m, 113 m^3
33. 17.5 cm, 22 463 cm^3
34. 2.0 cm, 50.8 cm^2
35. 3.0 mm, 112.4 mm^2
36. 10.8 cm, 1465.7 cm^2
37. 2.5 m, 80.7 m^2
38. $r = 4.2$, $V = 155.1$
39. $r = 6.3$, $V = 523.5$
40. $r = 2.8$, $V = 46.1$
41. $r = 10.6$, $V = 2492.3$
42. 65 kg
43. \$855
44. (a) 150.816 cm^3
    (b) 20
    (c) 823.68 cm^3
    (d) 16, No
45. (a) 16
    (b) 1018.008 cm^2
46. 73 800 cm^3, 11 640 cm^2
47. 2480 cm^3, 930 cm^2
48. 7507 cm^3, 2614 cm^2
49. (a) 103 cm^3
    (b) 52.4%
50. (a) 9.70 litres
    (b) 15 kg
51. (a) 12 cm (b) 18 cm
    (c) 1240 cm^3
52. 581.28 cm^3
53. (a) 37.2 cm^3, 0.524 cm^3
    (b) 0.7%
54. 93 692.6 cm^3

Chapter 8

1. 3, 0, –3
2. (a) 5, 7, 9
   (c) (i) 4, 6.4, 8.6
   (ii) –4, –2.2, –1
16. (b) 8
17. (b) 8
18. (b) 21
19. $x = 0$, $y = 2$
20. $x = 1$, $y = -1$
21. $x = -1$, $y = -2$
22. $x = 0$, $y = -2$
23. $x = 0.5$, $y = 1$
24. $x = 0.5$, $y = -3$
25. $x = 3.3$, $y = 1$
26. $x = 5$, $y = 0.4$
27. $x = 2.5$, $y = 4$
28. (0, 0)
29. (0, 0)

30. (0, 5) **31.** (0, –7)

32. (0, –4) **33.** $\left(0, \frac{1}{3}\right)$

34. (0, –4) **35.** $\left(0, \frac{1}{5}\right)$

36. (0, 0) **37.** (0, –2)

38. (0, –3) **39.** (0, –2)

40. $y = 2x + 2$

41. $y = -\frac{1}{2}x + 5$

42. $y = -3x - 4$

43. $y = \frac{2}{3}x - 2\frac{1}{2}$

44. $y = 9x - 1$

45. $y = 4x + 2.6$

46. $y = -4x - \frac{4}{5}$

47. $y = \frac{1}{5}x + 8$

48. $y = -\frac{2}{3}x - 7$

49. (0, 2)

50. $m = 4, c = -\frac{1}{2}$

51. l_1: $y = \frac{1}{2}x + 3$,

l_2: $y = \frac{1}{2}x + 4$,

l_3: $y = \frac{1}{2}x + 6$,

l_4: $y = \frac{1}{2}x - 6$,

l_5: $y = \frac{1}{2}x - 3$,

l_6: $y = \frac{1}{2}x - 2$

52. l_0: $y = -\frac{1}{3}x$,

l_2: $y = -\frac{1}{3}x + 4$.

l_3: $y = -\frac{1}{3}x + 6$,

l_4: $y = -\frac{1}{3}x - 8$,

l_5: $y = -\frac{1}{3}x - 5$,

l_6: $y = -\frac{1}{3}x - 3$,

l_7: $y = -\frac{1}{3}x - 2$

53. –1

54. **(a)** $y = 3x - 1$

(b) $b = 5$

55. **(a)** (0, 3) **(b)** $y = \frac{1}{3}x - 5$

56. **(a)** 15 units² **(b)** 4 units²

(c) 9 units² **(d)** 20 units²

57. **(a)** They have equal gradient but different constants. The lines are parallel.

(b) They have equal gradient and same constant. They are identical

Chapter 9

1. **(a)** 5, 2, 5, 10 **(c)** $x = 0$

(d) (0, 1)

2. **(a)** 5, –3, –4, –3 **(c)** $x = 2$

(d) (2, –4)

3. **(a)** 8, 9, 5 **(b)** $x = 3$

(c) (3, 9), maximum

4. **(a)** $a = 1, b = -1, c = 1$

(c) **(i)** 0.7, 4.3 **(ii)** 2.8

5. **(a)** $a = -4, b = -6$

(c) **(i)** 3.4

(ii) –3.2, 2.2

6. **(a)** 12, 6, –32

(c) **(i)** –0.2, 3.5

(ii) –0.8, 4.1

7. **(a)** 0, –8, –9, 0

(c) **(i)** –4, 2

(ii) –3.6, 1.6

(iii) –4.1, 2.1

(d) $x = -1, -9$

8. **(a)** $a = 4, b = 10, c = 4$

(c) **(i)** 6.3, 9, 3

(ii) –3.7, 2.7; –2.8, 1.8; –2.2, 1.2

(d) $v = -0.5, (-0.5, 10.3)$

9. **(a)** 7, –2, –8, –5

(c) 8 **(d)** –1.5, 3

10. **(a)** $A(-2, 0), B(0, -8), C(4, 0)$

(b) $x = 1, (1, -9)$

17. **(a)** **(i)** 0.3, 2.1

(ii) 0.5, 3; 1.5, 2

(b) 2.1, 1.8

18. **(a)** –1.3, 2.3 **(b)** 0.5

19. **(a)** –6.8 **(b)** –1.61, 5.61

20. **(a)** 11.5 **(b)** –4.39, 2.39

21. **(a)** $A(-2, 0), B(0, 12), C(2, 16), D(6, 0)$

(b) $x = 2$

22. **(a)** G_1: $y = -x^2$, G_2: $y = -x^2 - 2$

(b) G_3: $y = -x^2 - 4$, $x = 0$

(0, –4)

23. Case I

24. **(a)** –1, –2 **(b)** $x = -1.5$

(c) 2

25. (D)

26. **(a)** (4, 0) **(b)** –5

(c) $x = 2$, (2, 4) **(d)** 3, 1

Term III Revision Test

1. (2, –3)

2. 3, 5, –1

(a) $x = -0.5$ **(b)** 5.3

(c) 4

(d) 0.6 or –1.6

3. –10, –31, –26, 2

(a) –5.5

(b) 3.9 or –2.6

(c) 2.6 or –1.3

4. **(a)** **(i)** 217.4 m³

(ii) 2.2×10^8 cm³

(b) 64.4 m³

5. (2, 2)

6. **(a)** $-2, 1\frac{1}{2}, 1\frac{1}{2}, -2$

(c) **(i)** $t = 0$

(ii) 3.6

(iii) 2.8 or –2.8

7. **(a)** **(i)** 1754 cm³

(ii) $1165.68

(b) 81

8. **(a)** 470 000 mm³

(b) 0.00031 m³

(c) 155 cm³

9. **(a)** 8, 4, –2

(b) **(i)** 10.3

(ii) 2.7 or –3.7

(iii) 1.8

Chapter 10

1. **(a)** {6, 8, 10, 12, 14, 16, 18, 20, 22}

(b) {1, 4, 6, 8, 9, 10, 12, 14}
(c) {February}
(d) {Saturday, Sunday}
(e) {2, 3, 5, 7}
(f) {January, June, July}

2. (a) T (b) F (c) T
(d) F (e) F (f) F

3. (a) T (b) F (c) F
(d) F (e) T (f) T
(g) F (h) T

4. (a) T (b) T (c) T
(d) F (e) F (f) T
(g) T (h) T (i) F
(j) F

5. (a) T (b) T (c) F
(d) T (e) T (f) F
(g) T (h) F

6. (a) {7, 11, 13, 19, 23, 29, 31, 37, 41}
(b) {19, 23, 29}
(c) {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 43, 47}
(d) {2, 3, 5, 7, 11, 13, 17, 31, 37, 41, 43, 47}

7. (a) (i) 2 (ii) $\varnothing$
(b) 12, 14, 15, 16, 20, 22

8. (a) $\{x : -3.5 \leq x \leq 6\}$
(b) $\{x : -5 \leq x \leq -2 \text{ or } 3.5 < x \leq 10\}$
(c) $\{x : 3.5 < x \leq 6\}$

9. (a) {2, 4, 6, 8}
(b) {2, 4, 6, 8}

10. (a) {3}
(b) {21, 24, 27, 30, 33, 36, 39, 42, 45, 48}

11. $B = \{1, 3, 4, 5, 6, 8, 12\}$

12. (a) square or rhombus
(b) rectangle or parallelogram

13. (a) {25}
(b) {15, 25, 35}
(c) {13, 15, 16, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 36, 37, 39}

14. (a) $\{x : 4 < x \leq 10\}$
(b) $\{x : -1 < x < 8\}$
(c) $\{x : -1 < x \leq 4\}$
(d) $\{x : x \leq 4\}$

15. (a) 6 (b) 15 (c) 4

16. $B = \{0, 5, 10, 15, 20\}$,
$C = \{2, 3, 5, 7, 11, 13, 17, 19\}$

17. (a) (i) $A = \{5, 6, 7, 8, 9, 10, 11, 12, 13\}$,
(ii) $B = \{2, 5\}$
(b) {5} (c) 5

18. (a) $\subseteq$ (b) = (c) $\subseteq$
(d) $\subseteq$ (e) $\supseteq$ (f) $\in$
(g) $\notin$ (h) =

19. (a) 8 (b) c

20. (a) {2, 6, 8, 10, 12}
(b) {1, 9, 16}
(c) {3, 5, 7, 11, 13, 14, 15, 17, 18, 19, 20}
(d) (0, 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

21. (a) 5
(b) (i) {5, 7} (ii) {3, 9}

22. {(2, 5), (3, 5)}

23. (a) (i) $\{b, d, e, f\}$
(ii) $\{a, c, d, f, g\}$
(b) (i) 3 (ii) 5

24. (a) (i) $A = \{2, 3, 5, 7, 11, 13, 17, 19\}$,
(ii) $C = \{1, 2, 3, 4, 6, 12\}$
(b) 13, 6

25. (a) A (b) $\varnothing$ (c) B

26. (a) $A = B$

27. (a) $\{x : 24 \leq x \leq 32\}$
(b) $\{x : 21 \leq x \leq 36\}$

28. (a) {5, 6, 10, 15, 20}
(b) {5}

29. (a) {6, 8, 10, 12, 13, 14, 15 16, 18, 21}
(b) 2

30. (a) {2, 4, 8, 10, 14}
(b) 3

31. (a) $A = \{3, 4, 5, \ldots, 9\}$,
$B = \{2, 3, 4, \ldots, 10, 11\}$,
$C = \{3, 4, 5, \ldots, 17, 18\}$
(b) (i) 10 (ii) 17

32. (a) B (b) B'

33. (a) (i) {36, 81}
(ii) {21, 51, 81}
(b) 1

34. (a) {5, 7, 11} (b) 3

35. (a) 6 (b) 2 (c) 1
(d) 13 (e) 0 (f) 7

36. (a) P (b) $\varnothing$

37. (a) A (b) B

38. (a) {1, 2, 3, 4, 6}
(b) {1, 2, 3, 6}
(c) {1, 3}

39. (a) equilateral triangle
(b) rhombus
(c) square or rectangle

41. $a = 8, b = 10$;
$a = 14, b = 16$

42. (a) {(2, 2)}
(b) {(2, 4)}
(c) $\left\{\left(1\frac{1}{3}, 2\frac{2}{3}\right)\right\}$; ∞

Chapter 11

1. (a) 35 (b) 8.43
(c) 9 (d) $17\frac{1}{7}\%$

2. (a) 40 (b) 105.75
(c) 25

3. (a) 30 (b) $46\frac{2}{3}\%$
(c) 2 (d) 2.9

4. 14 marks

5. (a) (i) 1.8 m (ii) 1.8 m
(iii) 1.8 m
(b) 2.6 m

6. 13, 12.5, 11

7. 21.3, 20, 20

8. 13.9, 13.9, 12.6

9. 9.391, 9.4, 8.1

10. 17, 15

11. (a) 41 (b) 45 (c) 40

12. 28

13. (a) 64.9 seconds
(b) 65.2 seconds
(c) 65.15 seconds

14. 52

15. (a) $x = 10, y = 6$
(b) $x = 8, 3.3$

16. (a) 500 (b) $\frac{41}{100}$
(c) 2.05, 2, 1

17. (a)

No. of countries	0	1	2	3	4	5
Frequency	7	9	7	4	2	1

(b) 1, 1, 1.6

18. (a) 5, 2 (b) 12.5

19. (a) 3 (b) 2

(c) 2.3

20. (a) (i) 1 (ii) 3

(b) 1

21. 32, 37, 42, 47, 52, 57; 43 mm

22. (a) 45 (b) 56.75

23. 72.625 kg

24. 44.35 kg

25. (a) English 60, Chinese 67

(b) English 62, Chinese 66

(c) John did better and scored above average in English.

26. (a) 49

(b) Oct 26, Dec 37

(c) Oct 23.4, Dec 34.1

(d) More emergency calls in Dec, there are more than 30 calls per day in 20 days compared to only 10 days in Oct.

27. (b) Brand X (c) Brand Y

(d) Brand X is better, they last longer.

28. (a) 56 (b) 40.3 kg

(c) 60.3 kg (d) 46.4%

29. (a) 23 (b) 7

(c) 2.70

30. (a) 7 cm (b) 9 cm

(c) 50%

31. (a) 29 (b) 20 min

(c) 58.6%

(d) 8, 2, 0, 6, 5

32. (a) 7, 4, 4, 3, 1

(b) dot diagram

(c) 0, 2, 2.2

Chapter 12

1. (a) $\frac{1}{13}$ (b) $\frac{1}{52}$

(c) $\frac{1}{4}$ (d) $\frac{3}{13}$

2. (a) $\frac{1}{2}$ (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{5}{6}$

3. (a) (i) $\frac{3}{8}$ (ii) $\frac{5}{8}$

(b) (i) $\frac{3}{7}$ (ii) $\frac{4}{7}$

4. (a) (i) $\frac{3}{10}$ (ii) $\frac{7}{10}$

(b) (i) $\frac{4}{13}$ (ii) $\frac{9}{13}$

5. {HH, HT, TH, TT}

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

6. (a) (i) $\frac{1}{11}$ (ii) $\frac{3}{11}$

(iii) $\frac{4}{11}$ (iv) $\frac{7}{11}$

(b) (i) $\frac{1}{10}$ (ii) $\frac{1}{5}$

(iii) $\frac{3}{10}$ (iv) $\frac{7}{10}$

7. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$

8. (a) $\frac{1}{37}$ (b) $\frac{11}{37}$ (c) $\frac{19}{37}$

(d) $\frac{9}{37}$ (e) $\frac{18}{37}$

9. (a) (i) $\frac{1}{3}$ (ii) $\frac{17}{21}$

(iii) $\frac{2}{3}$

(b) (i) $\frac{3}{10}$ (ii) $\frac{4}{5}$

(iii) $\frac{7}{10}$

10. (a) $\frac{1}{2}$ (b) $\frac{1}{2}$

11. (a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{5}{8}$

12. (a) 14

(b) (i) $\frac{3}{10}$ (ii) $\frac{1}{2}$

(iii) $\frac{1}{5}$ (iv) $\frac{18}{25}$

(v) $\frac{7}{50}$ (vi) $\frac{17}{50}$

13. (a) (i) $\frac{14}{15}$ (ii) $\frac{1}{5}$

(iii) $\frac{8}{15}$ (iv) $\frac{8}{15}$

(b) (i) $\frac{3}{5}$ (ii) $\frac{1}{3}$

14. (a) $\frac{1}{2}$ (b) $\frac{3}{8}$ (c) $\frac{3}{8}$

15. (a) $x = 27$ (b) $\frac{3}{14}$

16. (a) $\frac{2}{15}$ (b) $\frac{13}{90}$

(c) $\frac{11}{45}$ (d) $\frac{1}{5}$

17. $\frac{31}{32}$; 50 bags

18. (a) $x = 48$

(b) (i) $\frac{3}{16}$ (ii) 0 (iii) $\frac{23}{64}$

19. (a) $x = 15$

(b) (i) $\frac{1}{6}$ (ii) 0 (iii) $\frac{11}{18}$

20. (a) $\frac{2}{25}$ (b) $\frac{4}{25}$ (c) $\frac{3}{25}$

21. $x = 20$

22. $x = 20$; $\frac{1}{3}$

23. $x = 10$

24. (a) (i) $\frac{1}{7}$ (ii) $\frac{3}{7}$

(b) $x = 14$

25. (a) $\frac{1}{13}$ (b) $\frac{12}{13}$

(c) $\frac{2}{13}$ (d) 0

26. (a) $\frac{7}{16}$ (b) $\frac{3}{8}$

(c) $\frac{7}{16}$ (d) 0

27. (a) $\frac{3}{7}$ (b) $\frac{17}{35}$

(c) $\frac{8}{35}$ (d) $\frac{11}{35}$

(e) $\frac{9}{35}$

28. (a) $\frac{17}{43}$ (b) $\frac{9}{43}$

(c) $\frac{4}{43}$ (d) $\frac{30}{43}$

Term IV Revision Test

1. (a) {11, 13, 17, 19, 23, 29}

(b) {15}

(c) {6, 9, 12, 15, 18, 21, 24, 27}

2. (a) {1, 2, 3, 4, 6, 8, 12}
   (b) {1, 2, 4, 6, 12}
   (c) 3
   (d) 3
3. $x = 6$
4. (a) $\frac{2}{5}$ (b) $\frac{9}{20}$ (c) $\frac{3}{10}$
5. (a) $4\frac{1}{2}$ (b) 4 (c) $3\frac{2}{3}$
6. (a) $r = \frac{8p + 15q}{3(p + 2q)}$
   (b) $2x^2y(x + 3y)(x - 3y)$
   (c) $1\frac{1}{2}$ or $-\frac{1}{2}$
7. 168.3 cm
8. (a) 30 min (b) 19, 29
9. (a) (i) 148 (ii) 2.96
   (b) (i) 2 (ii) 3
10. (b) the mode (c) 13, 13
11. (a) 33 min (b) 23 min
12. (a) 57 (b) 3.0 kg
    (c) 6.3 kg (d) 36.8%
13. (a) 7 (b) 5.67
14. (a) 80 (b) \$5200
    (c) 12, 11, 11.1

End-of-Year Specimen Paper

1. (a) $(a + b)(a - b)$
   (b) 160
2. 32
3. (a) (4, 0) (b) $x = 2$
   (c) $y = -4$, $x = 2$
4. (a) 4 cm (b) 10 cm
5. $x = \frac{3 + 5y}{y - 5}$
6. $13\frac{1}{3}$ cm
7. 3
8. $x = 7$, $y = -4$
9. (a) $\triangle ABC$ and $\triangle ADE$
   (b) $18\frac{2}{3}$
10. $5\frac{3}{13}$
11. 52 cm
12. (a) (i) {i}
    (ii) {a, e, i, o, u, z}
    (b) (i) 2 (ii) 4
13. 64
14. (a) $\frac{3}{10}$ (b) $\frac{1}{4}$ (c) 1
15. (a) $\frac{17}{40}$ (b) $\frac{9}{20}$
    (c) $\frac{1}{8}$ (d) $\frac{17}{40}$
16. (a) 30 (b) 105
17. $(2x + 1)^2 = (2x)^2 + (x - 1)^2$
    (a) 6 (b) 30 cm^2
18. $x = z$, $y = 4$
19. 62
20. (a) 265.94 cm^2
    (b) 407.77 cm^3
21. $9\frac{8}{45}$ cm
22. (a) $x^2 + y^2 = 2xy + 64$
    (b) 8
23. 5, 5, 3, −4
    (a) 6.1 (b) 3.9
    (c) 0.9 or −3.4
24. (a) 360 cm^2 (b) 400 cm^3
25. (a) (i) 2 (ii) 3
    (b) 3.4
26. (a) $300a + b = 2550$,
    $700a + b = 3150$
    (b) $a = 1\frac{1}{2}$, $b = 2100$
    (c) \$12
    (d) 500

New Syllabus

Mathematics Workbook 2 (Express)

New Syllabus Mathematics Workbook 2 (Express) is specially written to be used with New Syllabus Mathematics 2 Textbook. The chapters correspond with those in the textbook.

Contents in the workbook:

- Summary of Chapter
- Practice Questions
- Mindmap
- Alternative Assessment

In addition to the conventional questions that give students practice for their examinations, Alternative Assessment is included to provide students with new approaches to learning mathematics.

Alternative Assessment includes:

- Exploratory Worksheet
- IT Worksheet
- Mathematical Investigation
- Journal Writing
- Real-life Application
- Problem Posing

The combination of various pedagogies enables students to appreciate the mathematical concepts and to score well in the subject.